CONTROL OF A LIQUID LEVEL SYSTEM BASED ON A PROPORTIONAL-SUM CONTROLLER USING WHALE OPTIMIZER

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Abstract:

This study is about controlling the liquid level in water tanks, which is one of the challenges in the industry. A nonlinear analytical model is derived and validated against linearized continuous-time and discrete-time models, demonstrating their equivalence under nominal conditions. The classical method for tuning a discrete-time proportional-sum controller was applied, with gains tuned via the Ziegler-Nichols method. Additionally, the controller's parameters were fine-tuned by using the whale optimization algorithm. Simulation results for the tank system are presented. While Ziegler-Nichols gives a decent base, it can be said that optimization of the controller parameters should be recommended when dealing with similar problems in real-life situations. Results reveal that the optimized controller reduces the sum of squared errors compared to the classical controller, achieving superior accuracy.

Keywords:

Discrete-Time Systems, Whale Optimization Algorithm, Proportional-Difference-Sum Controller, Liquid Level Control, Ziegler-Nichols method.

INTRODUCTION

Control of liquid levels in tanks is always an important research topic, and a lot of scientific work has been done in order to find an optimal solution for this problem. The reason for this constant interest lies in the fact that tank plants can be found in multiple industries, like as chemical, pharmaceutical, oil and gas, water treatment, and so on.

When it comes to the utilization of PID-like controllers or their discrete-time equivalent proportional-difference-sum (PDS) controllers, the main task is to obtain PID parameters that provide good system behavior. Choosing proper parameters can significantly improve the performance of the system, while poor tuning can worsen it [1]. Authors in [1] offer a few conventional methods for tuning a PID controller. Although those methods may achieve the desired performance, the authors stated that a lot of effort and experience are needed for defining parameters. Intelligent controllers can also be used for this purpose.

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A classical fuzzy logic controller is implemented and compared to the PID controller in terms of obtaining the desired system response in [2]. Their result gave an advantage to the fuzzy controller. However, authors in paper [3] state in favor of PID controllers when it comes to their simple structure and easy tuning, and recommend using them for liquid level plants without substantial change in their dynamics. To regulate liquid levels under time-varying system behavior across diverse operational regimes, the authors employed a fuzzy-logic-based adaptive PID control scheme with gain scheduling. Artificial neural networks also found their application for solving this task. Paper [4] offers two different liquid level control strategies based on neural networks. The initial approach employs an inverse-model-driven neurocontroller, whereas the alternative strategy utilizes a neural-network-enhanced predictive control scheme. Reference [5] combines neural networks and fuzzy logic to propose a self-tuning neuro-fuzzy regulator designed for liquid level regulation, and it outperformed the fuzzy logic controller. The fusion of AI-based methods and conventional PID control can be achieved through evolutionary optimization strategies to adjust the PID gains. For example, in [6], for tuning PID controller parameters Genetic algorithm is used, and a comparison is made with PID tuned via the ZN method. As it was expected, including the metaheuristic algorithm brought better results. Sometimes, researchers modify the original metaheuristic algorithm, like in [7], where the Grey Wolf Optimization algorithm is modified for tuning PID. The increasing use of optimization algorithms in controlling systems is also reflected in their application in system modeling, like in [8], where the whale optimization algorithm (WOA) is employed to obtain an optimized Takagi-Sugeno plant model. Optimization algorithms are applicable in areas other than automatic control, such as in medicine [9], or in economics [10], where authors have used modified WOA to solve various tasks.

This work demonstrates liquid level control using a conventional proportional-sum (PS) controller whose parameters were optimized by the WOA [11]. The results are compared with the PS controller, whose parameters are obtained using the classical ZN tuning method [12].

2. SYSTEM CHARACTERIZATION

The system's physical characteristics employed in this study are presented in Table 1.

The water pump, reservoir, and two similar cylinder tanks, one above the other, make up the system. Water is pumped vertically from the reservoir to the upper tank through a pumping system. We need to control the water level of the second tank. The used system diagram is shown in Figure 1.

Table 1. Parameter configuration

Parts of the system	Labels	Numerical values	Unit of measurement
Pump flow constant	K_p	$5.37 \cdot 10^{-6}$	$m^3s^{-1}V^{-1}$
Diameter of outlet opening 1	D_{o1}	$0.47625 \cdot 10^{-2}$	m
Diameter of outlet opening 2	D_{o2}	$0.47625 \cdot 10^{-2}$	m
Tank 1 inside diameter	D_1	$4.445 \cdot 10^{-2}$	m
Tank 2 inside diameter	D_2	$4.445 \cdot 10^{-2}$	m
Gravitational constant	g	9.81	ms^{-2}

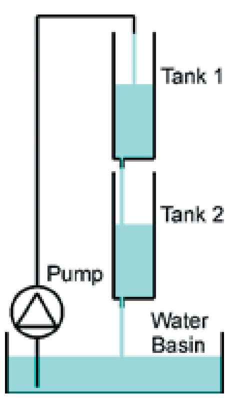

Figure 1. Liquid level system

3. MATHEMATICAL MODELING OF THE SYSTEM

3.1. ANALYTICAL NONLINEAR MODEL

Tank 1 and Tank 2 are the two subsystems of the plant. Pump voltage V_p is the input in tank 1, and tank 1 water level H_1 is the output. One way to express the flow into tank 1 is

$$Q_{i1}=K_p V_p .$$

Equation 1. Flow into tank 1

The outflow velocity V_{o1} and the opening cross-sectional area of tank 1, A_{o1}, are multiplied to give the outflow from tank 1,

$$Q_{o1}=A_{o1} V_{o1} .$$

Equation 2. Outflow from tank 1

The mass balance equation for tank 1 is

$$A_1 \frac{dH_1}{dt} = Q_{i1} - Q_{o1} = K_p V_p - A_{o1}\sqrt{2gH_1},$$

Equation 3. Mass balance equation for tank 1

where A_1 is the cross-sectional area of tank 1. Outflow velocity V_{o2} and the opening cross-sectional area of tank 2, A_{o2}, are multiplied to give the outflow from tank 2,

$$Q_{o2}=A_{o2} V_{o2} .$$

Equation 4. Outflow from tank 2

The mass conservation equation governing tank 2 is expressed as:

$$A_2 \frac{dH_2}{dt} = Q_{i2} - Q_{o2} = A_{o1}\sqrt{2gH_1} - A_{o2}\sqrt{2gH_2}.$$

Equation 5. Mass balance equation for tank 2

3.2. LINEAR MODELS

Since the water level in reservoir 2 is supposed to have a constant nominal value for steady-state conditions, the water level in reservoir 1 and the pump voltage also have constant values:

$$H_1=H_{1N},\ H_2=H_{2N},\ V_p=V_{pN}.$$

Equation 6. Nominal values for steady-state conditions

The next action to take is to use Taylor's series representation at nominal values, Equation 6, to approximate nonlinear functions given in Equation 7,

$$f_1 = \frac{dH_1}{dt} = \frac{K_p V_p}{A_1} - \frac{A_{o1}\sqrt{2gH_1}}{A_1}, \qquad (1)$$

$$f_2 = \frac{dH_2}{dt} = \frac{A_{o1}\sqrt{2gH_1}}{A_2} - \frac{A_{o2}\sqrt{2gH_2}}{A_2}. \qquad (2)$$

Equation 7. Nonlinear functions

As a result, the following linear differential equations are obtained:

$$\dot{h}_1=a_1 h_1+b_1 v_p, \qquad (1)$$

$$\dot{h}_2=a_2 h_2+b_2 h_1 . \qquad (2)$$

Equation 8. Linear differential equations

Variables h_1, h_2, and v_p in Equation 8 stand for deviations from nominal values:

$$h_1=H_1-H_{1N}, \qquad (1)$$

$$h_2=H_2-H_{2N}, \qquad (2)$$

$$v_p=V_p-V_{pN}, \qquad (3)$$

Equation 9. Deviations from nominal values

and coefficients a_1, a_2, b_1 and b_2 are calculated using the expressions below,

$$a_1 = -\frac{A_{o1} g}{A_1\sqrt{2gH_{1N}}},\ a_2 = -\frac{A_{o2} g}{A_2\sqrt{2gH_{2N}}}, \qquad (1)$$

$$b_1 = \frac{K_p}{A_1},\ b_2 = \frac{A_{o1} g}{A_2\sqrt{2gH_{1N}}}. \qquad (2)$$

Equation 10. Coefficients in linear differential equations

Based on Equation 10 and values from Table 1, linear continuous-time models in the form of transfer functions for the first and second reservoirs are easily determined by applying the Laplace transform to Equation 8.

For determining the linear discrete-time model, a sampling period of T = 0.01 s is adopted. The Zero Order Hold method is chosen as the discretization method.

Operating conditions and mathematical model representations (both continuous-time and discrete-time) for Tanks 1 and 2 are provided in Table 2.

Figure 2 represents an open-loop plant output comparison of nonlinear with linearized continuous-time and discrete-time models when the deviation from a nominal pump voltage equals v_p=0.4 V. As can be seen from Figure 2, the linear continuous-time and linear discrete-time models perfectly match, which allows for easier design of the controllers in the continuation of the work.

Table 2. Nominal operating conditions with dynamic models

	Tank 1	Tank 2
Nominal water level	H_{1N}=0.16 m	H_{2N}=0.16 m
Nominal pump voltage	V_{pN}=5.8775 V	
Linear continuous-time model	$G_1(s) = \frac{0.0544}{15.73s+1}$	$G_2(s) = \frac{1}{15.73s+1}$
Linear discrete-time model	$G_1(z) = \frac{3.4594 \cdot 10^{-5}}{z-0.9994}$	$G_2(z) = \frac{6.3540 \cdot 10^{-4}}{z-0.9994}$

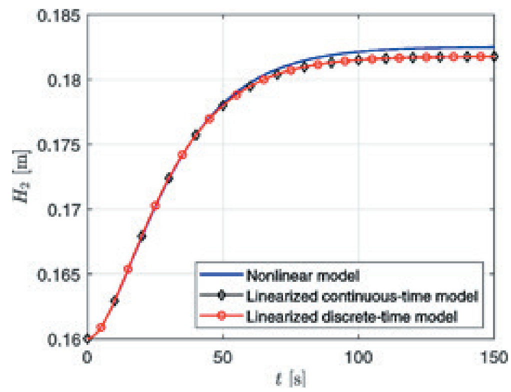

Figure 2. Continuous-time and discrete-time linear model accuracy against nonlinear plant behavior

4. CLASSICAL PI AND PS CONTROLLERS

Controller input is error signal $e(t)$,

$$e(t)=h_{2d}(t)-h_2(t),$$

Equation 11. Definition of error

and output is control signal $u(t)$ expressed with

$$u(t) = K_p\left(e(t) + \frac{1}{T_I}\int_0^t e(\tau)\mathrm{d}\tau\right) = K_P e(t) + K_I \int_0^t e(\tau)\mathrm{d}\tau .$$

Equation 12. PI control algorithm

PI controller s-transfer function equals

$$G_{PI}(s) = K_P\left(1 + \frac{1}{T_I s}\right).$$

Equation 13. PI controller s-transfer function

The proportional gain K_p and integral time constant T_I define the key parameters of the controller. Although the coefficients of PID-type controllers are adjustable through various tuning approaches, this study employs the established Ziegler-Nichols method to enable direct comparison with the metaheuristic optimization technique. These same controller parameters K_p and $K_I \equiv K_S$ can be applied in the discrete implementation of the zero-order proportional-sum controller, whose difference equation is described by:

$$u[k] = K_P e[k] + K_S T \sum_{j=0}^{j=k-1} e[j] .$$

Equation 14. Zero-order proportional-sum controller

By eliminating the sum from the previous equation, the final difference equation of the aforementioned PS controller is obtained,

$$u[k+1]=u[k]+K_p e[k+1]+(K_S T-K_p)e[k].$$

Equation 15. The final form of the difference equation for the PS controller

4.1. ZIEGLER-NICHOLS PI TUNING METHODOLOGY

The investigated plant represents a stable second-order system, making it suitable for the Ziegler-Nichols open-loop tuning method [12]. The procedure begins by obtaining the open-loop step response, which is then analyzed to extract the critical tuning parameters: dead time L, time constant T_1, and process gain K. These parameters are derived as illustrated in Figure 3. Table 3 provides the standard Ziegler-Nichols formulations used to compute the proportional and proportional-integral control parameters.

Table 3. PID tuning parameters calculated using the Ziegler-Nichols method

	K_P	T_I	T_D
P	$T_1/((KL))$	-	-
PI	$(0.9T_1)/((KL))$	$3.3L$	-
PID	$(1.2T_1)/((KL))$	$2L$	$0.5L$

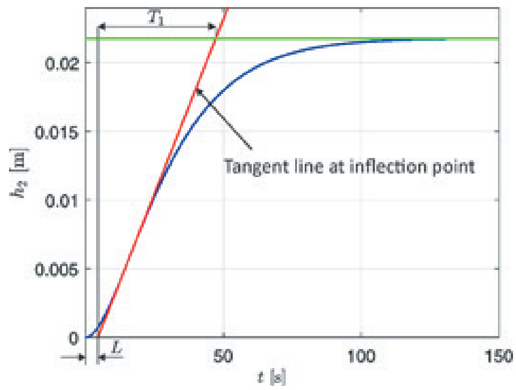

Figure 3. Tangent method

Lastly, the computed PS controller parameters obtained for the linear model are K_p=200.8541, K_S=16.9559.

5. THE WHALE OPTIMIZATION ALGORITHM

The whale-inspired optimization method has shown exceptional performance in addressing diverse nonlinear and multimodal challenges. The advantage of this method, and all metaheuristic algorithms in general, is the random distribution mechanism. This distribution helps avoid convergence to local minima. Proposed by Seyedali Mirjalili and Andrew Lewis in [11], WOA mimics the hunting behavior of humpback whales. It is the leader whale's responsibility to locate the fish. The remaining members track directional cues. In every hunt, they all take precisely the same position. They hunt in groups using a three-phase strategy: encircling prey, a bubble-net attack, and an adaptive search. The first phase is to determine the best search agent and update the positions of other agents. Using the distance vectors **D** and **X** to update the position, the mathematical model of this stage is:

$$\mathbf{A}=2\mathbf{ar}-\mathbf{a},\ \mathbf{C}=2\mathbf{r}, \quad (1)$$

$$\mathbf{D}=|\mathbf{C}\mathbf{X}'(t)-\mathbf{X}(t)|, \quad (2)$$

$$\mathbf{X}(t+1)=\mathbf{X}'(t)-\mathbf{AD}, \quad (3)$$

Equation 16. Mathematical model of the first phase

where **a** is linearly decreased from 2 to 0 and **r** is a random vector in [0, 1]. **A** and **C** are coefficient vectors, and t is the current iteration. **X** is the position vector, and **X'** is the position vector of the best solution so far. The fundamental mathematical models that mimic the second phase are the spiral path (first calculate the distance between the whale and prey using helix movement) and the shrinking encircling mechanism (define the new position of the searching agent using **A**). Between the original position and the best agent at the moment is the agent's new position. The function of this strategy is

$$\mathbf{X}(t+1)=\begin{cases}\mathbf{X}'(t)-\mathbf{AD} & \text{if} \quad p<0.5 \\ \mathbf{D}'e^{bl}\cos(2\pi l)+\mathbf{X}'(t) & \text{if} \quad p\geq 0.5' \end{cases}$$

Equation 17. Mathematical model of the second phase

where **D'** is the distance between the i-th whale and the prey, l is a random value in [-1, 1], b is a constant for the shape of the logarithmic spiral, and p is a random number in [0, 1]. In order to offer adequate connection between the first two phases, the third phase is based on the adaptive variation that relies on the value search vector **A**.

In this paper, with a population of 35 agents and 25 iterations (determined empirically via trial and error), each agent (whale) encodes a potential solution (optimal parameters K_p and K_S of the controller). Optimization minimizes the sum of squared errors (SSE) as the objective function,

$$\mathrm{SSE} = \sum_{i=1}^{n} e^2 [i] \, .$$

Equation 18. Objective function in the form of the sum of squared errors

Thirty independent runs were executed. A box plot of the objective function values across independent runs is shown in Figure 4 (left). The convergence graph of the best run is shown in Figure 4 (right).

The parameter values generated by the optimization algorithm are KP=299.8767, KS=1.9268.

6. SIMULATION RESULTS

This section presents simulation results demonstrating the plant's response under the control of the various controllers designed earlier. Figure 5 compares the level H_2 control performance between two distinct PS controller implementations. The whale optimization algorithm tunes PS controller gains (K_p and K_S) to outperform the classical Ziegler-Nichols method.

Figure 6 displays the change in control signals for each applied controller. To enhance plant performance, the proposed method generates a superior process input compared to Ziegler-Nichols PS control.

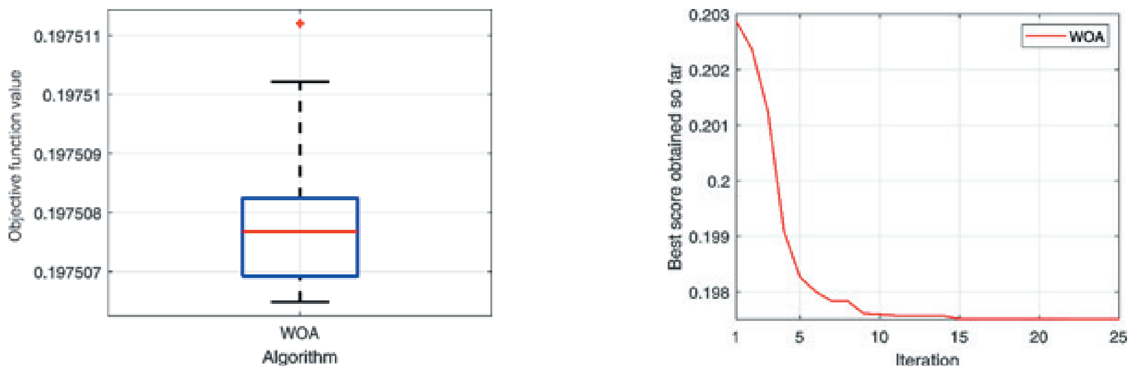

Figure 4. Box plot of 30-run fitness values (left) and convergence graph for the best execution (right)

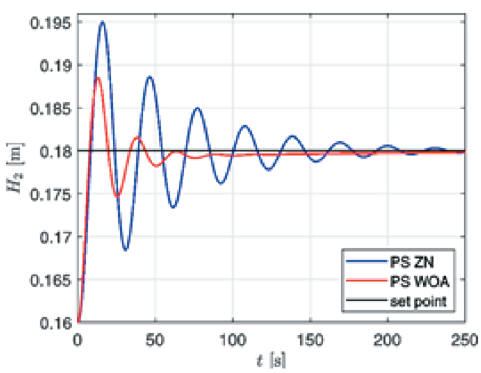

Figure 5. Closed-loop control of H_2 level using multiple PS controllers

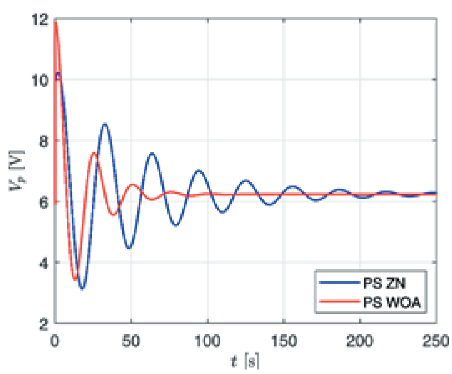

Figure 6. Simulated control signals for proportional-sum controllers

Table 4. Numerical values for classical and optimized PS controllers

Controller	SSE	Overshoot [%]	Settling time [s]
PS ZN	0.4839	75	157.03
PS WOA	0.1975	42.5	56.2

To quantitatively demonstrate the optimization's effectiveness, Table 4 presents the calculated sum of squared error values using Equation 18 for the chosen simulation duration. Additionally, in the same table, the system's dynamic behavior was given through both overshoot and settling time (±5% of the steady-state value). The comparative results in Table 4 indicate that the optimized PS controller outperforms the Ziegler-Nichols PS controller, as evidenced by the shown values.

7. CONCLUSION

In this paper, an analytical nonlinear model of a coupled tank system was obtained. After that, linear continuous-time and discrete-time models were determined. Their comparison was made to show that the linear models accurately describe the nonlinear system around the chosen nominal point. Subsequently, the coupled-tank liquid level process is regulated via a discrete-time proportional-sum control scheme. Gain values were identified employing the Ziegler-Nichols technique alongside the modern whale-inspired optimization approach. To assess the efficacy of their operation, the controllers were systematically compared. Performance comparison of all results was conducted using the minimum sum of squared errors criterion, percentage overshoot, and settling time values. This study highlights the efficacy of metaheuristic optimization in enhancing classical control methods for nonlinear industrial processes. Future studies will focus on the implementation of the obtained results on a real object under laboratory conditions, as well as on testing other optimization algorithms to further improve control performance.

8. ACKNOWLEDGMENTS

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