SPEED CONTROL OF A ROTARY SERVO-BASE UNIT: LYAPUNOV AND MIT RULE APPROACHES

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Abstract:

This paper presents a comparative study of two speed control techniques for a rotary servo-base unit using Lyapunov-based adaptive control and the MIT rule technique. The primary objective is to achieve precise and stable speed control and to analyze the influence of adaptation gain on the system performance. A mathematical model of the rotary system is analyzed, followed by the development of adaptive controllers based on the Lyapunov stability theory and the MIT rule. Choosing a suitable reference model is examined, and parameter adaptation laws are designed to optimize system performance. The impact of different adaptation gains on system response is evaluated through simulations in MATLAB/Simulink. Figures illustrating the evolution of adaptation parameters over time, as well as system response, are provided. Various performance criteria, settling time, overshoot, and different objective functions are used to compare the control approaches. The results highlight the advantages and limitations of each method. Recommendations for tuning adaptation parameters are provided to improve overall system performance.

Keywords:

Lyapunov Rule, MIT Rule, DC Motor, Model Reference Adaptive Control (MRAC), Adaptation Gain.

INTRODUCTION

Model reference adaptive control (MRAC) is an adaptive control strategy that develops a control law using an adjustable gain, making the system's plant continuously follow a reference model until the tracking error becomes zero [1]. The Massachusetts Institute of Technology (MIT) rule adjusts controller parameters using the gradient method based on the error between the plant's and reference model's output. However, a system created using the MIT rule can sometimes become unstable. In contrast, the Lyapunov approach guarantees stability by using the Lyapunov function that depends on the output and parameter error, ensuring the system remains stable as long as the derivative of the Lyapunov function is negative. Lyapunov and MIT rule approaches were compared in control of the coupled tank systems in the MRAC scheme in [2], or for the control of similar systems in [1], where fuzzy-optimized MRAC was applied.

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In this study, adaptive controllers are developed based on Lyapunov and MIT rules for speed control of DC motor, and a comparison between these two approaches is made.

2. DESCRIPTION OF THE SYSTEM

The creation of a mathematical model is among the first stages in the development of a control system. This saves time and profit in the long run [3]. In this section, we investigate the dynamics of a DC motor, which serves as the object in our control system. A full schematic representation of our object is shown in Figure 1.

2.1. ELECTRICAL DYNAMICS: VOLTAGE AND CURRENT EQUATIONS

The equations specifying the motor's electrical assemblies are listed below in Equation 1:

$$V_m(t) = R_m I_m(t) + L_m \frac{d}{dt} I_m(t) + e_b(t) \quad (1)$$

$$e_b(t) = k_m \omega_m(t) \quad (2)$$

Equation 1. Electrical dynamics of the object

Here: V_m, e_b, k_m, and ω_m are motor voltage, back electromotive voltage, back electromotive voltage constant, and speed of the motor shaft, respectively. Since the motor inductance L_m is much less than its resistance R_m, it can be ignored [4]. Solving the system of equations for motor current I_m, we get an electrical equation of a DC motor.

$$I_m(t) = \frac{V_m(t) - k_m \omega_m(t)}{R_m}$$

Equation 2. DC motor current

2.2. MECHANICAL DYNAMICS: THE SECOND NEWTON'S LAW OF MOTION

We consider the second Newton's law of motion and relationships between the following quantities: moment of inertia of the load J_l, of the motor shaft J_m; speed of the load shaft ω_l; viscous friction on both motor shaft B_m and the load shaft B_l. Total torques applied on the load τ_l and on the motor τ_m, with resulting torque acting on the motor shaft from the load torque denoted as τ_{ml}, are given with the following Equation 3:

$$J_l \frac{d\omega_l(t)}{dt} + B_l \omega_l(t) = \tau_l(t) \quad (1)$$

$$J_m \frac{d\omega_m(t)}{dt} + B_m \omega_m(t) + \tau_{ml}(t) = \tau_m(t) \quad (2)$$

Equation 3. Torque and motion analysis

Equation 4 represents the mechanical dynamics of the rotary servo base unit, which can be calculated with J_{eq} and B_{eq} as the total moment of inertia and damping term using the object's constants: η_g and K_g, which are the gearbox efficiency and the total gear ratio, respectively.

$$J_{eq} \frac{d\omega_l(t)}{dt} + B_{eq} \omega_l(t) = \eta_g K_g \tau_m(t)$$

Equation 4. Mechanical dynamics of the object

2.3. SYSTEM DYNAMICS INTEGRATION

Finally, when assuming that motor torque is proportional to voltage and with the introduction of the equivalent damping term $B_{eq,v}$, and the actuator gain A_m, consisting of motor efficiency η_m and current-torque constant k_t, the electromechanical model, Equation 5, is as follows:

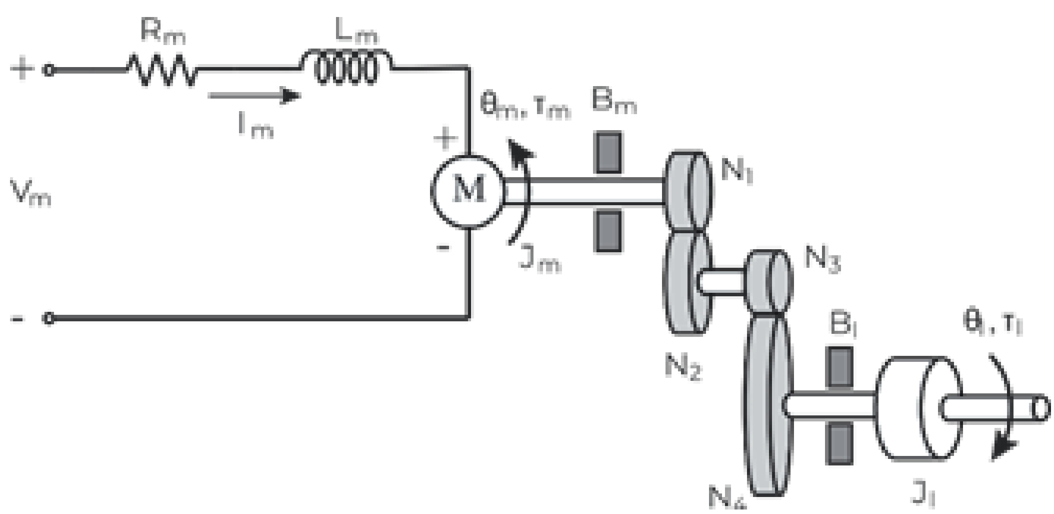

Figure 1. Illustration of the object under study

$$J_{eq}\left(\frac{d}{dt}\omega_l(t)\right) + B_{eq,v}\omega_l(t) = A_m V_m(t) \quad (1)$$

$$B_{eq,v} = \frac{\eta_g K_g^2 \eta_m k_t k_m + B_{eq} R_m}{R_m} \quad (2)$$

$$A_m = \frac{\eta_g K_g \eta_m k_t}{R_m} \quad (3)$$

Equation 5. Unified system representation

Choosing motor voltage as input $V_m(t)=u$, and angular speed of the load shaft as the output variable, $\omega_l(t)=y$ the system is now defined with J_{eq}=0.0021 kgm², $B_{eq,v}$=0.084 kgm² s⁻¹ and A_m= 0.1284Nm/V.

$$\dot{y} = -\frac{B_{eq,v}}{J_{eq}}y + \frac{A_m}{J_{eq}}u$$

Equation 6. State equation of the object

The system's block diagram is given in Figure 2.

3. MODEL REFERENCE ADAPTIVE CONTROL (MRAC)

Figure 3 displays the structural diagram of a typical MRAC system. It has two loops: one for parameter modification and one for feedback. In this diagram, y_d is the set point or reference, y_m is the reference model's output, y is the object's output, and u is the input (and control law which depends on the adaptation parameters) in the object.

To adjust the control algorithm and make the object track the reference model's output, y_m, the controller's parameters are tuned. The adaptation law or adjustment mechanism of the MRAC system can be found using a variety of methods. The gradient approach, also known as the MIT rule, a stability theory the Lyapunov method, or some others can all be used to carry out the MRAC adjustment mechanism [5].

3.1. LYAPUNOV RULE

Since the plant of interest in this paper is the first-order object, Equation 6, we will consider a first-order plant and a reference model given by all positive coefficients (a, b, a_m, b_m) and control algorithm.

$$\frac{dy}{dt} = -ay + bu \quad (1)$$

$$\frac{dy_m}{dt} = -a_m y_m + b_m y_d \quad (2)$$

$$u = \theta_1 y_d - \theta_2 y \quad (3)$$

Equation 7. First-order adaptive control structure

When we substitute Equation 7(3) into Equation 7(1) and take the derivative of the error function, which we define as the difference between the real output of the plant, y, and the output of the reference model, y_m, we obtain:

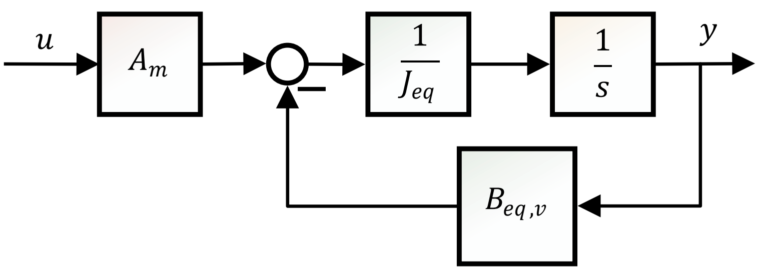

Figure 2. Block diagram of a linear system

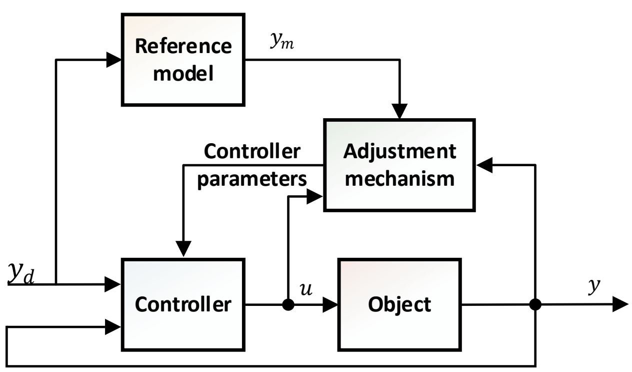

Figure 3. Structural diagram of a general idea for MRAC

$$e = y - y_m \quad (1)$$
$$\dot{e} = -a_m e - (b\theta_2 + a - a_m)y + (b\theta_1 - b_m)y_d \quad (2)$$

Equation 8. The error function and its derivation

The algorithms for changing the parameters in the MRAC system can be explained by the Lyapunov stability theory. A Lyapunov function, V, is necessary for the Lyapunov method. It must be positive definite, and its derivative, $\dot{V}$, must be negative definite. Since there is no methodical approach to determining an appropriate Lyapunov function [6], the Lyapunov function and its derivative are chosen as:

$$V(e,\theta_1,\theta_2) = \frac{1}{2}\left(e^2 + \frac{1}{b\gamma}(b\theta_2 + a - a_m)^2 + \frac{1}{b\gamma}(b\theta_1 - b_m)^2\right) \quad (1)$$
$$\dot{V} = -a_m e^2 + \frac{1}{\gamma}(b\theta_2 + a - a_m)(\dot{\theta}_2 - \gamma y e) + \cdots \quad (2)$$
$$\ldots + \frac{1}{\gamma}(b\theta_1 - b_m)(\dot{\theta}_1 + \gamma y_d e)$$

Equation 9. Lyapunov function with derivative

with positive γ. For $\dot{V}$ to be negative definite, we will cancel the second and third terms in Equation 9(2) by ensuring that the adjustable parameters are updated as:

$$\frac{d\theta_1}{dt} = -\gamma y_d e \quad (1)$$
$$\frac{d\theta_2}{dt} = \gamma y e \quad (2)$$

Equation 10. Lyapunov rule adaptation

where γ represents the tuning parameter (or adaptation parameter, adaptation gain). These two equations from Equation 10 are Lyapunov adjusting mechanisms I and II. They are shown graphically in Figure 4.

3.2. THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY (MIT) RULE

Derivation of the MIT rule is also achieved using the error function from Equation 8 and objective function $J(\boldsymbol{\theta}) = \frac{1}{2}e^2$ with its derivative $\frac{\partial J}{\partial e} = e$. The goal is adjusting the parameter $\boldsymbol{\theta} = [\theta_1\ \theta_2]$ so that direction of the negative gradient of the objective function is guaranteed with the adaptation gain γ_1, which is utilized to modify the controller's adaption rate: $\frac{d\boldsymbol{\theta}}{dt} = -\gamma^1 \frac{\partial J}{\partial \boldsymbol{\theta}} = -\gamma^1 e \frac{\partial e}{\partial \boldsymbol{\theta}} \rightarrow \frac{d\theta_1}{dt} = -\gamma^1 e \frac{\partial e}{\partial \theta_1}$ and $\frac{d\theta_2}{dt} = -\gamma^1 e \frac{\partial e}{\partial \theta_2}$. Similar to the first step in the previous section, when we insert the third equation of Equation 7 into the first and compare the actual and reference models, taking into account that $y_m = y$ (or at least asymptotically trace), we get:

$$\theta_1 = \frac{b_m}{b} \quad (1)$$
$$\theta_2 = \frac{a_m - a}{b} \quad (2)$$

Equation 11. Control parameters in MIT rule using reference model

Taking the Laplace transform of the first two equations of Equation 7 (with substituting the third equation into the first) and inserting them into the definition of the error, Equation 8(1), the following equations are obtained:

$$e(s) = \frac{b\theta_1}{s + a + b\theta_2} y_d - \frac{b_m}{s + a_m} y_d \quad (1)$$
$$\frac{\partial e}{\partial \theta_1} = \frac{b}{s + a + b\theta_2} y_d \overset{Equation\ 11(2)}{=} \frac{b}{s + a_m} y_d \quad (2)$$
$$\frac{\partial e}{\partial \theta_2} = -\frac{b^2\theta_1}{(s + a + b\theta_2)^2} y_d \overset{Equation\ 11(2)}{=} -\frac{b}{s + a_m} y \quad (3)$$

Equation 12. Partial error derivation

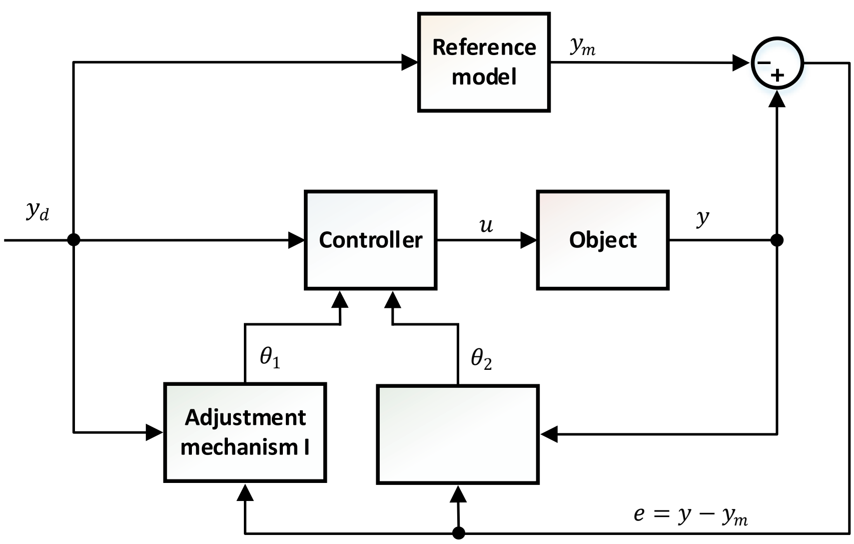

Figure 4. MRAC structural diagram using the Lyapunov rule

Finally, the MIT rule-based adaptation laws include:

$$\frac{d\theta_1}{dt} = -\gamma e \frac{a_m}{s+a_m} y_d \qquad (1)$$

$$\frac{d\theta_2}{dt} = \gamma e \frac{a_m}{s+a_m} y \qquad (2)$$

Equation 13. MIT rule adaptation,

where $\gamma = \frac{\gamma^{1} b}{a_m}$. It is found that the adaptation laws derived from the Lyapunov and MIT rules are alike (see Figure 5), with the exception that the MIT rule adds a filter that is equal to the reference model's transfer function [7].

4. RESULTS

The reference model used in this paper is in the very simple transfer form: $W_{ref} = \frac{1}{0.02s+1}$ In Figure 6, results for the reference speed of $y_d = 1\,\frac{rad}{s}$ are presented for different adaptation gains γ. The adaptive gain impacts stability and convergence, with larger gains causing faster response times but potentially large overshoots and oscillations, while smaller values improve stability but slow the system response.

In general, the properties of the control system and the performance requirements determine which adaptation gain should be used.

Comparison and the system's dynamic behavior quality for both rules are given in Table 1 and in Figure 7. Since responses for γ = 0.01 and γ = 0.1 do not reach the reference in 4s, the overshoot and the settling time (±2% of the steady-state value) are not calculated for them. Four different objective functions are calculated: Integral Square Error (ISE), Integral Absolute Error (IAE), Integral Time-weighted Absolute Error (ITAE), and Integral Time-weighted Square Error (ITSE). Results show, both in Table 1 and Figure 7, that higher values of the adaptation parameter guarantee smaller errors with shorter settling times.

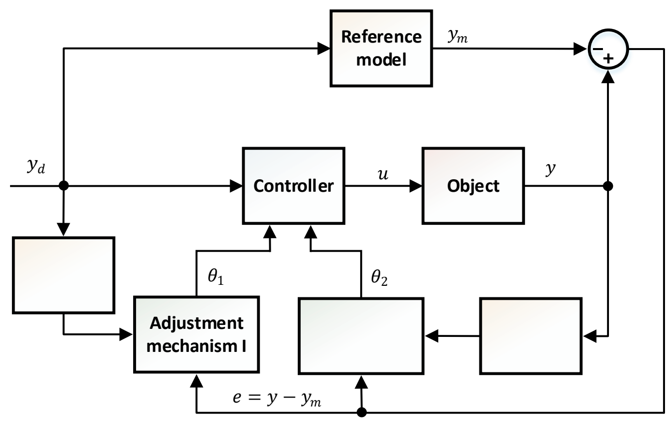

Figure 5. MRAC structural diagram using the MIT rule

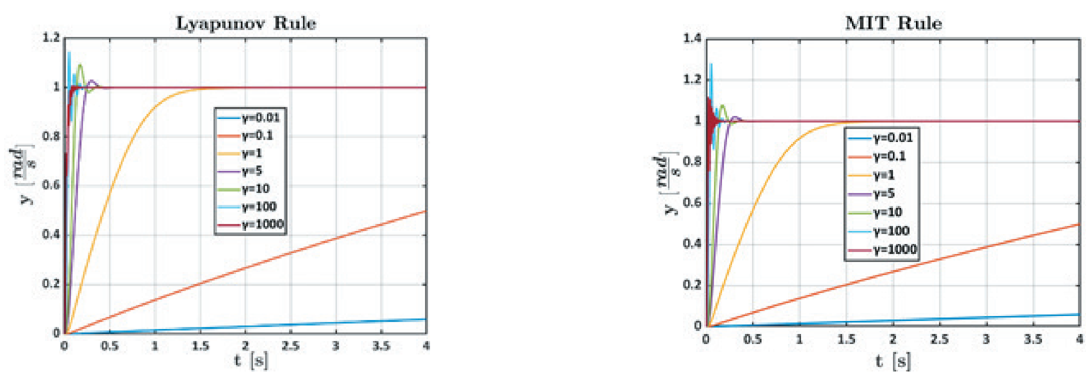

Figure 6. Simulation results for the Lyapunov rule (left) and MIT rule (right)

Table 1. Comparison and measurement quality

	Lyapunov rule						MIT rule					
Adaptation gain	Overshoot [%]	Settling time [s]	Objective functions				Overshoot [%]	Settling time [s]	Objective functions			
			ISE	IAE	ITAE	ITSE			ISE	IAE	ITAE	ITSE
0.01	/	/	0.4668	0.4784	0.1240	0.1232	/	/	0.4788	0.4864	0.1243	0.1236
0.1	/	/	0.4513	0.4723	0.1194	0.1142	/	/	0.4513	0.4723	0.1194	0.1142
1	0	1.2979	0.2470	0.3413	0.0772	0.0500	0	1.3098	0.2580	0.3489	0.0774	0.0503
5	2.59	0.3273	0.0607	0.1050	0.0096	0.0044	1.97	0.2445	0.0715	0.1132	0.0098	0.0048
10	9.30	0.2183	0.0327	0.0647	0.0045	0.0016	7.74	0.2202	0.0427	0.0715	0.0045	0.0019
100	14.40	0.1456	0.0042	0.0183	8.7E-4	1.1E-4	28.24	0.1478	0.0098	0.0263	0.0011	2.4E-4
1000	0.78	0.0879	4.1E-4	0.0051	1.9E-4	1.0E-5	11.89	0.0988	0.0021	0.0105	3.4E-4	4.0E-5

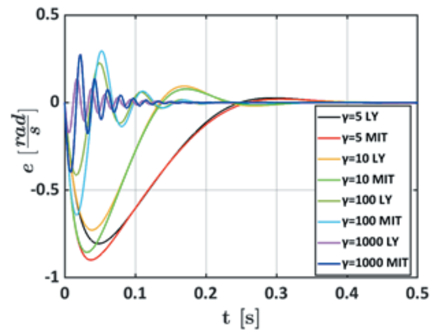

Figure 7. Error signal for Lyapunov and MIT rules for different γ values

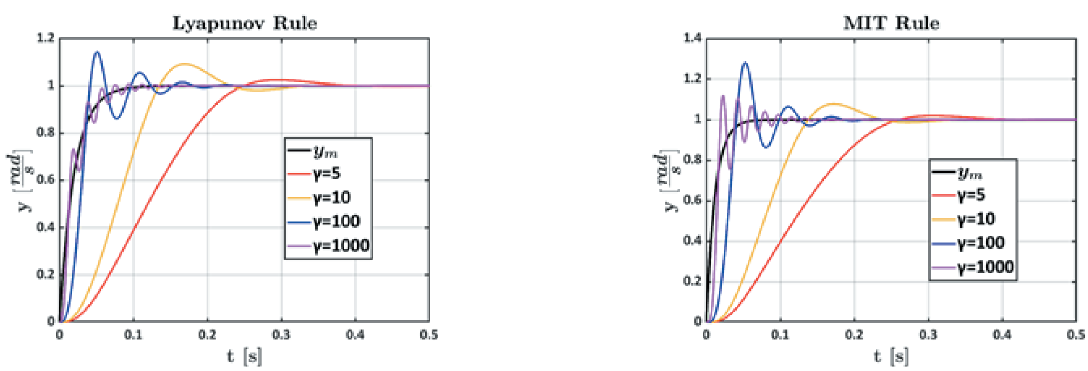

Figure 8. Difference between reference model and plant output for Lyapunov (left) and MIT rule (right)

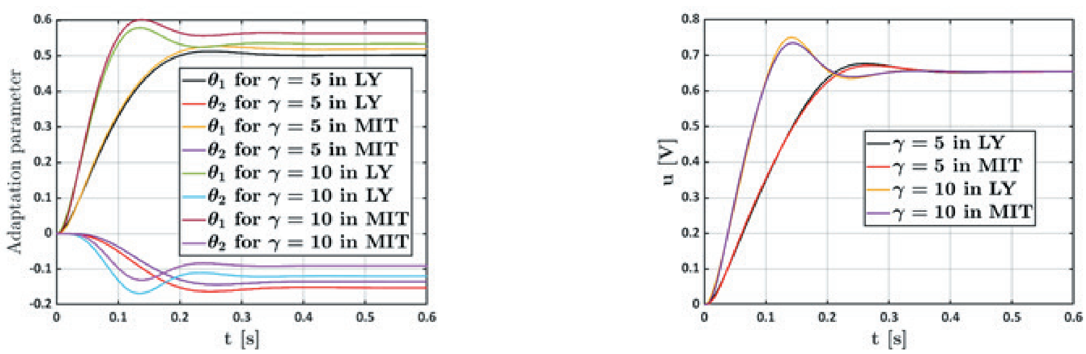

Figure 9. Changes in the parameters of adaptation θ_1 and θ_2 (left) and in the control signals (right)

The following Figure 8 shows the difference between output reference model y_m and plant output y for the Lyapunov and MIT rules only for γ>1. Changes in the Lyapunov (Equation 10) and in MIT (Equation 13) rule adaptation are shown in the following Figure 9 (left) for two different tuning parameters: γ = 5 and γ = 10. For the same tuning parameters, Figure 9 (right) shows the control signal.

For further testing, a special function was designed to generate different reference motor speeds. The function is given in the form of the time-dependent piecewise constant function.

Figure 10 shows the response of the system for the reference speed of the mentioned constant piecewise function as the reference output for both the Lyapunov and MIT rules. That higher values of the adaptive gain proved to be a better choice is confirmed also in Figure 11, where the sinusoidal signal was selected as the reference. For both the Lyapunov and MIT techniques, figures for system response and control signals appear nearly identical, with differences not explicitly visible.

5. CONCLUSION

In this study, the Lyapunov and MIT adaptive control algorithms were investigated and applied to control the speed of a rotary servo-base unit. A first-order transfer function was created as the reference model to ensure a smooth system response and match with the dynamics of the real object. The time constant of the chosen reference model is 0.02s, which enables the DC motor to track the reference signal with a minimal delay while avoiding oscillations or overshoots. The trial-and-error approach was applied to determine the adaptation gain γ, leading to the conclusion that higher values (above 1) result in improved system performance. This paper offers practical guidelines for selecting the reference model and the adaptation gain in model reference adaptive control applications. For future research, more advanced approaches, such as machine learning methods and fuzzy-neural or metaheuristic optimization algorithms, could be employed to find out the reference model and adaptation parameters in the control system.

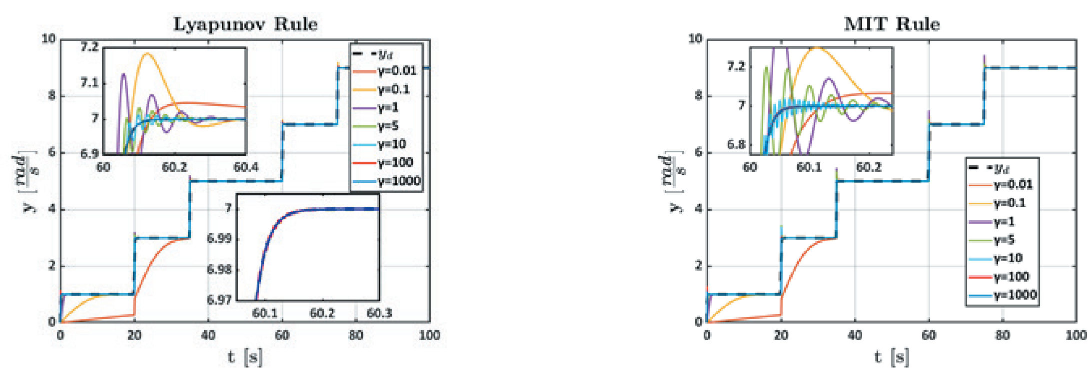

Figure 10. Simulation results for step changeable reference signal

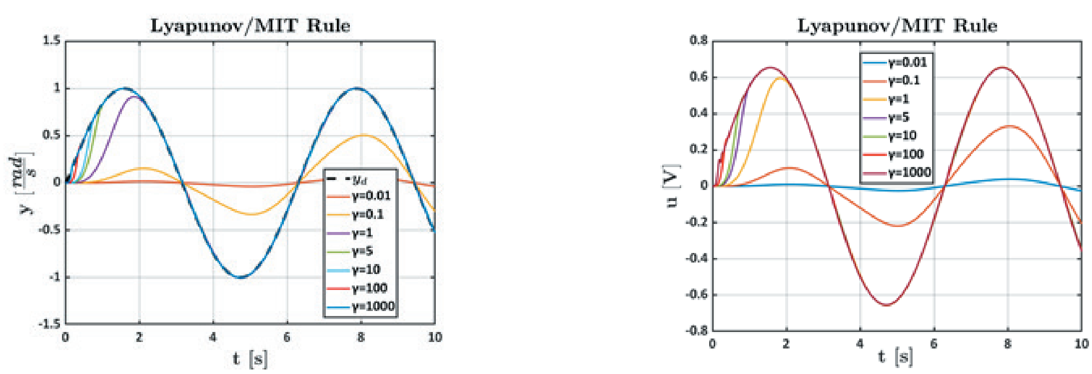

Figure 11. Simulation results for a sinusoidal reference signal (left) and control signal (right)

6. ACKNOWLEDGEMENTS

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