



FORECASTING BASE METALS PRICES: A COMPARISON OF VARIOUS BAYESIAN-BASED METHODS

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Abstract:

This paper addresses the topic of forecasting base metal prices index using advanced Bayesian methods, emphasising Bayesian dynamic mixture models. Original schemes were expanded by certain modifications. A broad set of macroeconomic indicators, such as interest rates, industrial production, economic activity, market stress indices, others commodities prices, exchange rates and information from stock markets, etc. were taken as potential predictors. Models were recursively estimated, taking under consideration possible discrepancy between released and revised data, carefully simulating real-time forecasting conditions. Dynamic Model Averaging was found to provide the highest accuracy of predictions compared to competing models. The forecasts were significantly more accurate than the ARIMA method or the no-change method. Among the dynamic mixture variants, model selection appeared to offer the best performance. The Clark-West test for nested models confirmed that forecast combination schemes lead to significant forecast accuracy improvements. Sector companies' stock prices and particular exchange rates were found to be the important base metals price predictors.

Keywords:

Bayesian Dynamic Mixture Models, Dynamic Model Averaging, Forecasting Accuracy, Model Averaging, Model Selection.

INTRODUCTION

Forecasting metal prices has become an important area of focus in economic research. Over time, there has been a shift from traditional econometric methods to more advanced techniques that promise greater prediction accuracy. Precise forecasting of metal prices is vital for various stakeholders, such as policymakers, investors, and industries that rely on raw materials. Metal prices are affected by numerous factors, including supply and demand fluctuations, geopolitical developments, and financial market conditions, which makes predicting them a complex task. Recently, the adoption of advanced methods, especially machine learning models, has gained traction in the field of metal price forecasting [1-5]. The aim of this research is to provide an insight into this topic from Bayesian dynamic mixture models (BDMM) applied to forecasting the monthly World Bank index of metal prices [6]. Contrary to traditional models often failing to account for non-linear relationships and structural breaks caused by unpredictable events, such as the COVID-19 pandemic or geopolitical tensions, BDMM offers a promising technique [7-8].

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2. DATA

Monthly data between 09/2002 and 10/2024 were analysed. World Bank Base Metals Price Index (2010 = 100, USD), including aluminium, copper, lead, nickel, tin and zinc was taken as the dependent variable. Similarly, as in previous studies [1-5], and, for example, Buncic and Moretto [9] the following variables were taken as independent ones: Consumer Price Index for All Urban Consumers: All Items in U.S. City Average (CPIAUCSL); U.S. 3-month risk-free rate (monthly averages, 3-month treasury bill: secondary market rate, TB3MS), a proxy of short-term interest rate; U.S. 10-year government bond yields (monthly averages, IRLTLT01USM156N), a proxy of long term interest rate; U.S industrial production (INDPRO); Kilian Index of Global Real Economic Activity (IGREA); term spread (10-Year Treasury Constant Maturity Minus 2-Year Treasury Constant Maturity for U.S.; monthly averages, T10Y2YM); St. Louis Fed Financial Stress Index (monthly averages, STLFSI4); implied volatility VIX index (monthly averages); The Caldara and Iacoviello GPR index; S&P 500 index (\wedge SPX); Dow Jones Industrial index (\wedge DJI); Shanghai Composite Index (\wedge SHC); MSCI EM index for emerging markets; monthly average prices of gold (USD per troy ounce); monthly average WTI oil price (USD per barrel). Additionally, exchange rates (to USD) of the largest producers of aluminium, lead, nickel and zinc (both primary metal and ore) were taken, i.e., China (USDCNY), Russia (USDRUB), India (USDINR), Indonesia (CCUSSP02IDM650N and since 01/2024 US-DIDR), South Korea (USDKRW), Australia (AUDUSD), Guinea (monthly averages, GNF) and Philippines (US-DPHP). Also, largest metal companies share prices were taken, i.e., Rio Tinto Plc (RIO.UK, in GBX), Alcoa Corp (AA.US, in USD), Hindustan Zinc Limited (HINDZINC.BO, in INR), Teck Resources Limited (TECK, in USD), BHP Group Limited (BHP, in USD), and Sherritt International Corporation (S.TO, in CAD) [6, 10-22].

If not otherwise stated, the last observations in a period were taken. All variables, except interest rates, Kilian index, term spread, financial stress index, VIX and GPR were included in log-differences. For computational reasons, and stationarity issues, the variables were later standardized based on a mean and standard deviation of the first 100 observations (Table 1 and Table 2). This value was kept further as the in-sample. However, obtained forecasts were transformed back to the level values, and then evaluated. To mimic real-market data availability, all independent variables were lagged one period back, and the Kilian index was lagged two periods back. Initially, the data as released in the past were taken, i.e., not in a revised form [23].

3. METHODS

All models were estimated recursively, i.e., each forecast at time t was done over all the data available up to time $t-1$. The following models were estimated: Bayesian dynamic mixture (BDMM) with state space (SS) and normal regression components (NR). The original schemes [7-8] were additionally improved by model averaging (A) and model selection schemes (H) [24]. They were denoted by BDMM-SS, BDMM-SS-A, BDMM-SS-H, BDMM-NR and BDMM-NR-H respectively. Additionally, Dynamic Model Averaging (DMA) and Dynamic Model Selection (DMS) were estimated (with the standard forgetting factors equal to 0.99). Bayesian Model Averaging (BMA) and Bayesian Model Selection (BMS) were also estimated [25]. These models were also estimated in versions with component models being just single variable ones (i.e., simple linear regression of one independent variable and a constant). They were denoted by DMA-1VAR, DMS-1VAR, BMA-1VAR and BMS-1VAR, respectively [26].

LASSO, RIDGE and Elastic Net (EL-NET) regressions were estimated, both conventionally and in their Bayesian versions, i.e., B-LASSO and B-RIDGE. Following the recursive approach, the penalty parameter was chosen by the t -fold cross-validation based on the Mean Square Error (MSE) metric. Mixing parameters $\{0.1, 0.2, \dots, 0.9\}$ were employed. Additionally, the least-angle regression (LARS) was estimated [27-29].

Time-varying parameters regressions, both with forgetting (TVP-FOR) and without (TVP) were estimated [26]. ARIMA was estimated with automatic lag selection [30]. The no-change (NAÏVE) method, a historical average over all past observations (HA) and over rolling window of last 100 observations (HA-ROLL) were also estimated.

In addition, when suitable, variance updating was done as by Raftery et al. [25], and additionally the version with the exponentially weighted moving average was estimated with parameter $\kappa = 0.97$ as recommended, for example, in [31]. Such version of models was denoted by adding “-K”, i.e., BDMM-SS-K, BDMM-SS-A-K, BDMM-SS-H-K, DMA-K, DMS-K, DMA-1VAR-K, DMS-1VAR-K, BMA-K, BMS-K, BMA-1VAR-K, BMS-1VAR-K, TVP-K and TVP-FOR-K. Secondly, BDMM-NR-1MOD denotes the model BDMM-NR where the only component is the model with all considered independent variables. In the case of BDMM-SS and BMA such a reduction results in the TVP model, and for DMA – in TVP-FOR model [26].



Table 1. Descriptive statistics

	Min	Max	Mean	Median	Standard Deviation	Coefficient of Variation	Skewness
p_metals	-4.2048	2.1591	-0.1208	-0.1038	0.7820	-6.4713	-0.8220
cpi	-4.8579	2.8450	0.0230	-0.0025	0.8613	37.4467	-0.6132
r_short	-1.1416	1.9272	-0.2644	-0.6292	1.0223	-3.8662	0.9867
r_long	-5.3409	1.7318	-1.5594	-1.7494	1.7734	-1.1372	-0.0638
ind_prod	-8.0029	1.9768	0.0960	0.1892	0.7306	7.6086	-6.7462
ec_act	-4.2736	2.0267	-1.1602	-1.2689	1.2785	-1.1019	0.3432
term_spread	-2.3669	1.3546	-0.3392	-0.2884	0.9734	-2.8695	-0.0524
fin_stress	-0.8724	4.8788	-0.2141	-0.4022	0.6875	-3.2116	3.7128
VIX	-1.0883	4.0797	-0.1901	-0.3819	0.8051	-4.2355	2.3035
GPR	-0.7115	5.7244	0.1868	0.0023	0.8288	4.4357	2.8961
SP500	-4.1981	2.5353	0.0614	0.1757	0.9446	15.3745	-0.7754
DJ_Ind	-3.6352	2.9383	0.0453	0.1269	0.9665	21.3151	-0.5550
SSE	-3.1314	2.5712	-0.0318	-0.0151	0.7847	-24.7042	-0.4833
MSCI	-4.6947	1.9523	-0.1250	-0.1160	0.8331	-6.6672	-0.8312
p_gold	-3.3274	2.3270	-0.1542	-0.2054	0.8840	-5.7330	-0.0410
p_oil	-6.4683	5.7477	-0.0826	0.0619	1.1254	-13.6287	-0.9461
fx_CNY	-3.1334	4.6269	0.1768	0.1050	1.2344	6.9823	0.5422
fx_RUB	-5.4840	7.6815	0.1817	0.0160	1.7863	9.8299	0.8612
fx_INR	-3.6819	4.1998	0.1285	0.0430	1.0303	8.0189	0.2759
fx_IDR	-3.3691	5.1459	0.0662	0.0581	0.8740	13.2031	0.9473
fx_KRW	-3.9837	3.3854	0.0337	-0.0188	0.8094	24.0421	0.1544
fx_AUD	-4.4667	2.3168	-0.1323	-0.0929	0.8666	-6.5500	-0.5371
fx_GNF	-6.3337	3.2157	-0.1344	-0.2190	0.6782	-5.0449	-2.0818
fx_PHP	-2.2100	2.6953	0.1191	0.0638	0.9346	7.8471	0.2739
Rio_Tinto	-5.0927	2.1439	-0.0625	-0.0789	0.8114	-12.9911	-1.0748
Alcoa	-6.3662	3.4061	0.0121	0.0586	1.1152	92.1101	-1.1646
Hindustan_Zinc	-2.4213	3.6508	-0.1208	-0.1596	0.7362	-6.0935	0.8719
Teck	-5.5518	3.0392	-0.0918	-0.1007	0.8403	-9.1537	-0.9763
BHP	-3.3694	2.5499	-0.1436	-0.1179	0.9520	-6.6297	-0.2834
Sherritt	-5.4259	4.0323	-0.1377	-0.1582	1.1137	-8.0889	-0.4420



Table 2. Stationarity tests. (For all tests 0.0100 indicates value less than 0.01. For Kwiatkowski-Phillips-Schmidt-Shin test 0.1000 indicates value greater than 0.1.)

	augmented Dickey-Fuller test statistic	augmented Dickey-Fuller test p-value	Phillips- Perron test statistic	Phillips- Perron test p-value	Kwiatkowski- Phillips- Schmidt-Shin test statistic	Kwiatkowski- Phillips- Schmidt-Shin test p-value	Skewness
p_metals	-6.2814	0.0100	-173.3586	0.0100	0.1324	0.1000	-0.8220
Cpi	-4.8201	0.0100	-133.1739	0.0100	0.3016	0.1000	-0.6132
r_short	-2.6648	0.2959	-2.8191	0.9420	0.5390	0.0329	0.9867
r_long	-1.0857	0.9234	-6.2037	0.7623	1.8374	0.0100	-0.0638
ind_prod	-7.1646	0.0100	-228.9841	0.0100	0.2105	0.1000	-6.7462
ec_act	-2.8170	0.2318	-22.9402	0.0369	1.7232	0.0100	0.3432
term_spread	-2.2485	0.4713	-6.6974	0.7346	1.4911	0.0100	-0.0524
fin_stress	-3.3587	0.0618	-31.1382	0.0100	0.3217	0.1000	3.7128
VIX	-3.4809	0.0450	-41.8297	0.0100	0.1830	0.1000	2.3035
GPR	-7.0251	0.0100	-64.7243	0.0100	0.7558	0.0100	2.8961
SP500	-5.7640	0.0100	-257.3397	0.0100	0.1015	0.1000	-0.7754
DJ_Ind	-5.6475	0.0100	-253.0829	0.0100	0.0644	0.1000	-0.5550
SSE	-4.7156	0.0100	-271.5288	0.0100	0.0420	0.1000	-0.4833
MSCI	-6.1156	0.0100	-245.3847	0.0100	0.2399	0.1000	-0.8312
p_gold	-5.6370	0.0100	-221.1622	0.0100	0.2435	0.1000	-0.0410
p_oil	-6.5283	0.0100	-168.1905	0.0100	0.0775	0.1000	-0.9461
fx_CNY	-6.5491	0.0100	-254.2642	0.0100	0.3558	0.0962	0.5422
fx_RUB	-6.6391	0.0100	-195.8817	0.0100	0.1502	0.1000	0.8612
fx_INR	-5.5961	0.0100	-233.7721	0.0100	0.1668	0.1000	0.2759
fx_IDR	-6.6932	0.0100	-238.7224	0.0100	0.0577	0.1000	0.9473
fx_KRW	-6.3087	0.0100	-270.5470	0.0100	0.1184	0.1000	0.1544
fx_AUD	-6.5947	0.0100	-258.6252	0.0100	0.2818	0.1000	-0.5371
fx_GNF	-5.8932	0.0100	-137.3176	0.0100	0.3462	0.1000	-2.0818
fx_PHP	-6.1184	0.0100	-248.4363	0.0100	0.2890	0.1000	0.2739
Rio_Tinto	-6.7902	0.0100	-249.9814	0.0100	0.0558	0.1000	-1.0748
Alcoa	-5.7666	0.0100	-279.7361	0.0100	0.0475	0.1000	-1.1646
Hindustan_Zinc	-5.8495	0.0100	-230.7092	0.0100	0.3541	0.0970	0.8719
Teck	-7.3485	0.0100	-236.2602	0.0100	0.0777	0.1000	-0.9763
BHP	-6.3720	0.0100	-266.9019	0.0100	0.2585	0.1000	-0.2834
Sherritt	-6.4427	0.0100	-240.5759	0.0100	0.1512	0.1000	-0.4420



Herein, $K=29$ independent variables are considered. In case of BDMM schemes and DMA, DMS, BMA and BMS, all possible multilinear regression models (i.e., 2^K , because the constant-only model is also included) are originally considered as component models. This leads to serious computational obstacles. Therefore, DMA-1VAR, DMS-1VAR, BMA-1VAR and BMS-1VAR models were estimated, as they are based on $K+1$ models only (the model with constant only is included). The second approach to reduce the number of component models is as follows. The set of independent variables is split into interest rates, economic activity, market stress, stock markets, other commodities prices, exchange rates, and metal sector stock price factors, i.e.,

$$x_1 = [\text{cpi}, r_{\text{short}}, r_{\text{long}}];$$

$$x_2 = [\text{ind}_{\text{prod}}, \text{ec}_{\text{act}}];$$

$$x_3 = [\text{term}_{\text{spread}}, \text{fin}_{\text{stress}}, \text{VIX}, \text{GPR}];$$

$$x_4 = [\text{SP500}, \text{DJ}_{\text{Ind}}, \text{SSE}, \text{MSCI}];$$

$$x_5 = [\text{p}_{\text{gold}}, \text{p}_{\text{oil}}];$$

$$x_6 = [\text{fx}_{\text{CNY}}, \text{fx}_{\text{RUB}}, \text{fx}_{\text{INR}}, \text{fx}_{\text{IDR}}, \text{fx}_{\text{KRW}}, \text{fx}_{\text{AUD}}, \text{fx}_{\text{GNF}}, \text{fx}_{\text{PHP}}]; \text{ and}$$

$$x_7 = [\text{Rio}_{\text{Tinto}}, \text{Alcoa}, \text{Hindustan}_{\text{Zinc}}, \text{Teck}, \text{BHP}, \text{Sherritt}].$$

Then, all possible multilinear regression models are constructed for each set of independent variables, i.e., for x_1, x_2, \dots, x_7 . Finally, the model with all independent variables is added. As a result, instead of 2^{29} , just $1 + 2^3 + 2^2 + 2^4 + 2^4 + 2^2 + 2^8 + 2^6 - 7 + 1 = 363$ component models must be considered, which corresponds to less than 9 independent variables for the original scheme, and which is computationally feasible. Simultaneously, such a split and combinations are economically reasonable, as they keep forecast averaging idea of the modelling schemes, and emphasise different economic groups of factors possibly influencing metals prices.

4. RESULTS

Table 3 presents forecast accuracy metrics of the estimated models. Root Mean Square Error (RMSE), normalized RMSE (N-RMSE), Mean Absolute Error (MAE) and Mean Absolute Scaled Error (MASE) were computed [32]. According to all metrics, DMA-K is the most accurate method, followed by DMA. Despite the poor performance of BDMM models, several of these schemes outperform NAÏVE or ARIMA. Out of these schemes, BDMM-NR-H performs the best. If revised data is taken, outcomes are quite comparable [23]. Indeed, for the robustness of results, initially, the models

were estimated with released data, mimicking real-time forecasting. However, versions with revised data (as of 12/2024) were also estimated [10-11, 23].

According to the Diebold-Mariano test [33] with 10% significance level and squared error loss function, forecasts from DMA-K are significantly more accurate than those from NAÏVE and ARIMA, TVP, TVP-FOR, TVP-K and TVP-FOR-K, as well as, many other models, but not from BDMM-SS-A-K or BDMM-NR-H. On the other hand, BDMM-NR-H forecasts more accurately than NAÏVE, but not than ARIMA. It forecasts more accurately than BDMM-NR-1MOD, and more accurately than many other models, but not as much as DMA-K. If revised data is taken, outcomes are similar, but BDMM-NR-H is found to be additionally more accurate than ARIMA. (Due to the limited space detailed outcomes are not presented herein.) Moreover, the Model Confidence Set test [34], at a 5% significance level, eliminated BDMM-SS, BDMM-SS-K, TVP, HA and HA-ROLL models.

When forecasts from models with released data were compared with those based on revised data with the Diebold-Mariano test with a 5% significance level, both with squared error and absolute scaled error loss functions, different accuracy can be assumed for BDMM-SS, RIDGE and TVP. For EL-NET, B-LASSO, B-RIDGE and TVP-K different accuracy can be assumed only when absolute scaled error loss functions are applied.

The Giacomini-Rossi fluctuation test over approximately 2.75-year periods, at a 5% significance level, does not indicate that BDMM-NR-H forecasting performance would perform worse than DMA-K [35].

The Clark-West test for nested models was performed [36]. Only in two cases, assuming 5% significance level, the larger model cannot be said to generate smaller errors than the restricted (simple) model. This provides some evidence that model combination schemes provide significant gains to increase forecast accuracy. Details are in Table 4.

Finally, in case of variable selection, DMA-K ascribed the highest weights to share prices, whereas BDMM-NR-H also did so, but it ascribed even higher weights to exchange rate variables.



Table 3. Forecast accuracy metrics

	RMSE	N-RMSE	MAE	MASE
BDMM-SS	8.9581	0.0945	6.6615	2.1327
BDMM-SS-A	4.2094	0.0444	3.1482	1.0079
BDMM-SS-H	4.4507	0.0470	3.3030	1.0575
BDMM-SS-K	8.8260	0.0931	6.6613	2.1327
BDMM-SS-A-K	3.9377	0.0416	2.9582	0.9471
BDMM-SS-H-K	4.0204	0.0424	3.1099	0.9957
BDMM-NR	4.1208	0.0435	3.0920	0.9899
BDMM-NR-H	3.8553	0.0407	2.9276	0.9373
BDMM-NR-1MOD	4.2338	0.0447	3.1337	1.0033
DMA	3.7607	0.0397	2.8458	0.9111
DMS	3.8380	0.0405	2.9448	0.9428
DMA-1VAR	3.9542	0.0417	2.9751	0.9525
DMS-1VAR	4.0874	0.0431	3.0604	0.9798
BMA	3.8074	0.0402	2.9053	0.9301
BMS	3.9057	0.0412	3.0041	0.9618
BMA-1VAR	4.0663	0.0429	3.0733	0.9839
BMS-1VAR	4.0714	0.0430	3.0697	0.9828
DMA-K	3.7462	0.0395	2.8108	0.8999
DMS-K	3.8248	0.0404	2.8649	0.9172
DMA-1VAR-K	3.9090	0.0412	2.9452	0.9429
DMS-1VAR-K	3.8988	0.0411	2.9262	0.9369
BMA-K	3.8167	0.0403	2.8613	0.9161
BMS-K	3.8460	0.0406	2.8940	0.9265
BMA-1VAR-K	4.0183	0.0424	2.9975	0.9597
BMS-1VAR-K	4.0180	0.0424	2.9963	0.9593
LASSO	3.9114	0.0413	2.9295	0.9379
RIDGE	3.8333	0.0405	2.8919	0.9259
EL-NET	3.8448	0.0406	2.9002	0.9285
B-LASSO	3.8463	0.0406	2.8938	0.9265
B-RIDGE	3.9209	0.0414	2.9417	0.9418
LARS	3.9901	0.0421	3.0171	0.9659
TVP	5.2093	0.0550	3.9573	1.2670
TVP-FOR	4.2491	0.0448	3.2028	1.0254
TVP-K	4.1956	0.0443	3.0911	0.9896
TVP-FOR-K	4.2250	0.0446	3.1473	1.0076
ARIMA	4.0074	0.0423	3.0326	0.9709
NAIVE	4.1940	0.0443	3.1235	1.0000
HA	20.1314	0.2124	15.2861	4.8939
HA-ROLL	20.1629	0.2128	15.8311	5.0684



Table 4. The Clark-West test outcomes

larger	null	CW statistic	CW p-value
BDMM-SS	TVP	1.3743	0.0847
BDMM-SS-A	TVP	5.4306	0.0000
BDMM-SS-H	TVP	5.1591	0.0000
BDMM-SS-K	TVP-K	-1.1000	0.8643
BDMM-SS-A-K	TVP-K	4.4544	0.0000
BDMM-SS-H-K	TVP-K	3.8545	0.0001
BDMM-NR	BDMM-NR-1MOD	4.5229	0.0000
BDMM-NR-H	BDMM-NR-1MOD	4.6307	0.0000
DMA	TVP-FOR	5.2999	0.0000
DMS	TVP-FOR	5.2130	0.0000
BMA	TVP	6.5274	0.0000
BMS	TVP	6.5215	0.0000
DMA-K	TVP-FOR-K	4.0591	0.0000
DMS-K	TVP-FOR-K	3.7423	0.0001
BMA-K	TVP-K	3.5581	0.0002
BMS-K	TVP-K	3.4871	0.0002

5. CONCLUSIONS

Forecasting base metal prices index with various Bayesian-based methods, focusing on Bayesian dynamic mixture models was discussed, both in original versions and with further improvements. A comprehensive large set of economic indicators was applied, and models were estimated recursively, mimicking real-time forecasting conditions. Dynamic Model Averaging was found to be the most accurate forecasting scheme, whereas out of Bayesian dynamic mixture models, the scheme with normal regression components and selection was the most accurate. The proposed methods outperformed ARIMA or no-change forecast. For robustness, models based on released data were compared with those based on revised data. According to the applied statistical tests, there is a gain in forecast accuracy from applying more advanced model combination schemes over simple models.

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Republishing the full data is unavailable due to copyright restrictions. The majority of the data applied in the study are openly available as cited in the text (suitable tickers provided) and in References. The forecast data generated in the study are openly available in Figshare at <https://doi.org/10.6084/m9.figshare.28382480>.



REFERENCES

- [1] H. Zhang, H. Nguyen, D.-A. Vu, X.-N. Bui, B. Pradhan, "Forecasting monthly copper price: A comparative study of various machine learning-based methods," *Resources Policy*, vol. 73, p. 102189, 2021. Available: <https://doi.org/10.1016/j.resourpol.2021.102189>
- [2] P. Pincheira Brown, N. Hardy, "Forecasting base metal prices with the Chilean exchange rate," *Resources Policy*, vol. 62, pp. 256-281, 2019. Available: <https://doi.org/10.1016/j.resourpol.2019.02.019>
- [3] T. Shi, C. Li, W. Zhang, Y. Zhang, "Forecasting on metal resource spot settlement price: New evidence from the machine learning model," *Resources Policy*, vol. 81, p. 103360, 2023. Available: <https://doi.org/10.1016/j.resourpol.2023.103360>
- [4] H. Guo, J. Wang, Z. Li, H. Lu, L. Zhang, "A non-ferrous metal price ensemble prediction system based on innovative combined kernel extreme learning machine and chaos theory," *Resources Policy*, vol. 79, p. 102975, 2022. Available: <https://doi.org/10.1016/j.resourpol.2022.102975>
- [5] H. Ben Ameer, S. Boubaker, Z. Ftihi, et al., "Forecasting commodity prices: Empirical evidence using deep learning tools," *Annals of Operations Research*, vol. 339, pp. 349-367, 2024. Available: <https://doi.org/10.1007/s10479-022-05076-6>
- [6] World Bank, "Commodity markets." Accessed Feb. 3, 2025. [Online.]: Available: <https://www.worldbank.org/en/research/commodity-markets>
- [7] I. Nagy, E. Suzdaleva, "Mixture estimation with state-space components and Markov model of switching," *Applied Mathematical Modelling*, vol. 37, pp. 9970-9984, 2013. Available: <https://doi.org/10.1016/j.apm.2013.05.038>
- [8] I. Nagy, E. Suzdaleva, M. Karny, T. Mlynarova, "Bayesian estimation of dynamic finite mixtures," *International Journal of Adaptive Control and Signal Processing*, vol. 25, pp. 765-787, 2011. Available: <https://doi.org/10.1002/acs.1239>
- [9] D. Buncic, C. Moretto, "Forecasting copper prices with dynamic averaging and selection models," *The North American Journal of Economics and Finance*, vol. 33, pp. 1-38, 2015. Available: <https://doi.org/10.1016/j.najef.2015.03.002>
- [10] ALFRED, "Archival FRED." Accessed Feb. 3, 2025. [Online.]: Available: <https://alfred.stlouisfed.org>
- [11] FRED, "Economic data." Accessed Feb. 3, 2025. [Online.]: Available: <https://fred.stlouisfed.org>
- [12] L. Kilian, "Not all oil price shocks are alike: Disentangling demand and supply shocks in the crude oil market," *American Economic Review*, vol. 99, pp. 1053-1069, 2009. Available: <https://doi.org/10.1257/aer.99.3.1053>
- [13] Chicago Board Options Exchange. (2025). Historical data [Online]. Available: https://www.cboe.com/tradable_products/vix/vix_historical_data
- [14] D. Caldara, M. Iacoviello, "Measuring geopolitical risk," *American Economic Review*, vol. 112, no. 4, pp. 1194-1225, 2022. Available: <https://doi.org/10.1257/aer.20191823>
- [15] M. Iacoviello. (2025). Geopolitical risk (GPR) index [Online]. Available: <https://www.matteoiacoviello.com/gpr.htm>
- [16] Stooq, "Stooq." Accessed Feb. 3, 2025. [Online.]: Available: <https://stooq.pl/index.html>
- [17] MSCI, "End of day index data search." Accessed Feb. 3, 2025. [Online.]: Available: <https://www.msci.com/end-of-day-data-search>
- [18] FXTOP, "Historical rates." Accessed Feb. 3, 2025. [Online.]: Available: <https://fxtop.com/en/historical-exchange-rates.php>
- [19] Yahoo, "Yahoo!finance." Accessed Feb. 3, 2025. [Online.]: Available: <https://finance.yahoo.com>
- [20] P. Shrivastava, R. Vidhi, "Pathway to sustainability in the mining industry: A case study of Alcoa and Rio Tinto," *Resources*, vol. 9, p. 70, 2020. Available: <https://doi.org/10.3390/resources9060070>
- [21] J. J. Barry, G. R. Matos, W. D. Menzie, "U.S. Mineral Dependence - Statistical Compilation of U.S. and World Mineral Production, Consumption, and Trade, 1990-2010," Open-File Report 2013-1184, U.S. Geological Survey: Reston, VA, 2013. Available: <https://pubs.usgs.gov/of/2013/1184>
- [22] N. E. Idoine, E. R. Raycraft, S. F. Hobbs, P. Everett, E. J. Evans, A. J. Mills, D. Currie, S. Horn, R. A. Shaw, "World Mineral Production 2018-22," British Geological Survey: Keyworth, Nottingham, 2024. Available: <https://nora.nerc.ac.uk/id/eprint/537241/1/World%20Mineral%20Production%202018%20to%202022.pdf>
- [23] D. Croushore, "Frontiers of real-time data analysis," *Journal of Economic Literature*, vol. 49, pp. 72-100, 2011. Available: <https://doi.org/10.1257/jel.49.1.72>
- [24] K. Drachal, "dynmix': An R package for the estimation of dynamic finite mixtures," *SoftwareX*, vol. 22, p. 101388, 2023. Available: <https://doi.org/10.1016/j.softx.2023.101388>
- [25] A. E. Raftery, M. Karny, P. Ettler, "Online prediction under model uncertainty via dynamic model averaging: Application to a cold rolling mill," *Technometrics*, vol. 52, pp. 52-66, 2010. Available: <https://doi.org/10.1198/TECH.2009.08104>
- [26] K. Drachal, "Dynamic Model Averaging in economics and finance with fdMA: A package for R," *Signals*, vol. 1, pp. 47-99, 2020. Available: <https://doi.org/10.3390/signals1010004>



- [27] J. Friedman, T. Hastie, R. Tibshirani, "Regularization paths for generalized linear models via coordinate descent," *Journal of Statistical Software*, vol. 33, pp. 1–22, 2010. Available: <https://doi.org/10.18637/jss.v033.i01>
- [28] R. B. Gramacy. (2019). monomvn: Estimation for MVN and Student-t Data with Monotone Missingness [Online]. Available: <https://CRAN.R-project.org/package=monomvn>
- [29] T. Hastie, B. Efron. (2013). lars: Least Angle Regression, Lasso and Forward Stagewise [Online]. Available: <https://CRAN.R-project.org/package=lars>
- [30] R. J. Hyndman, Y. Khandakar, "Automatic time series forecasting: The forecast package for R," *Journal of Statistical Software*, vol. 26, pp. 1–22, 2008. Available: <https://doi.org/10.18637/jss.v027.i03>
- [31] G. Koop, D. Korobilis, "Forecasting inflation using dynamic model averaging," *International Economic Review*, vol. 53, pp. 867–886, 2012. Available: <https://doi.org/10.1111/j.1468-2354.2012.00704.x>
- [32] R. J. Hyndman, A. B. Koehler, "Another look at measures of forecast accuracy," *International Journal of Forecasting*, vol. 22, pp. 679–688, 2006. Available: <https://doi.org/10.1016/j.ijforecast.2006.03.001>
- [33] F. X. Diebold, R. S. Mariano, "Comparing predictive accuracy," *Journal of Business and Economic Statistics*, vol. 13, pp. 253–263, 1995. Available: <https://www.jstor.org/stable/1392155>
- [34] P. R. Hansen, A. Lunde, J. Nason, "The model confidence set," *Econometrica*, vol. 79, pp. 453–497, 2011. Available: <https://doi.org/10.3982/ECTA5771>
- [35] R. Giacomini, B. Rossi, "Forecast comparisons in unstable environments," *Journal of Applied Econometrics*, vol. 25, pp. 595–620, 2010. Available: <https://doi.org/10.1002/jae.1177>
- [36] T. E. Clark, K. D. West, "Approximately normal tests for equal predictive accuracy in nested models," *Journal of Econometrics*, vol. 138, pp. 291–311, 2007. Available: <https://doi.org/10.1016/j.jeconom.2006.05.023>