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ADVANCED TECHNOLOGIES AND APPLICATIONS SESSION

STATISTICAL MODELLING OF ATMOSPHERIC TURBULENCE IN FREE-SPACE OPTICAL COMMUNICATION SYSTEMS

Nenad Stanojević[1]*,
[0009-0004-4158-6412]
Đoko Banđur[1],
[0000-0001-9034-6854]
Đorđe Šarčević[2],
[0000-0003-0746-744X]
Petar Spalević[1],
[0000-0002-6867-7259]
Stefan Panić[3]
[0000-0002-5868-1764]

[1]Faculty of Technical Sciences,
University of Priština – Kosovska Mitrovica,
Kosovska Mitrovica, Serbia

[2]Department of Medical and Bussines-Technological Studies,
Academy of Professional Studies Šabac,
Šabac, Serbia

[3]Faculty of Sciences, University of Priština – Kosovska Mitrovica,
Kosovska Mitrovica, Serbia

Abstract:

The paper focuses on the statistical modeling of atmospheric turbulence, a crucial aspect in predicting signal performance. Three distributions—Gamma-Gamma, Rician, and Nakagami-m play a significant role in understanding and forecasting the influence of atmospheric conditions on Free-Space Optical (FSO) communication systems. The Gamma-Gamma distribution provides insights into scintillation effects, the Rician distribution addresses scenarios with a dominant line-of-sight component, while the Nakagami-m distribution is employed to model situations with multiple scatters. In the part of the paper exploring the Gamma-Gamma distribution, various forms of the distribution are graphically represented depending on the strength of turbulence. The spreading of the distribution, serving as an indicator of signal degradation, is straightforwardly depicted based on the turbulence strength. In the case of the Rician distribution, a derived expression for the Probability Density Function (PDF) is provided relative to the K factor. Additionally, the distribution's relationship is visually represented as the K factor increases, illustrating the relationship between the dominant component and the scattering-induced component. In the section dedicated to the Nakagami-m model, an analysis of the properties of this distribution is conducted depending on the fading parameter (m). By examining the characteristics of the distributions presented in this study, the selection process for modeling systems is facilitated. Despite the drawbacks associated with FSO, the statistical modeling of turbulence provides valuable insights for optimizing system performance in real-world scenarios.

Keywords:

FSO, atmospheric turbulence, Gamma-Gamma distribution, Rician Distribution, Nakagami-m distribution.

INTRODUCTION

Optical fibers have a crucial role in transmitting information through broadband internet networks. In telecommunications, optical fibers have become the standard infrastructure for transmitting information. Optical fibers, with their wide bandwidth and low losses, are employed for signal transmission over significant distances. The implementation of optical fibers is further facilitated by their relatively affordable cost and compatibility of components with wireless communication systems. To overcome obstacles, it is often the case that these systems are combined with some wireless communication systems.

Correspondence:
Nenad Stanojević

e-mail:
nenads25@gmail.com

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Literature [1] mentions wireless optical communications as a significant factor in connecting end-users to the optical fiber system, serving as the final segment of the connected network, commonly known as the last mile. Urban environments and inaccessible terrains can pose particular challenges, where the construction of optical systems may take a considerable amount of time, presenting difficulties for rapid network expansion. Free-Space Optical (FSO) systems, as part of optical wireless solutions, present a good solution for overcoming the last-mile problem. These systems can function as an alternative or complement to optical fibers. During signal transmission interruptions in optical fibers, FSO systems can act as a temporary bypass. In paper [2], the FSO system is introduced as the prevailing solution for the last-mile challenge in commercial metro network systems between fixed locations. The paper outlines the benefits of implementing the FSO system while also discussing its limitations in terms of applicability. Furthermore, potential solutions for overcoming these obstacles are presented in the paper. In paper [3], the rationale for implementing FSO systems, their ease of deployment, and cost-effectiveness are outlined. The architecture of FSO systems is analyzed, along with factors contributing to the widespread adoption of this technology. Additionally, the effects of rain and fog on FSO signal propagation are examined. Suggestions are also provided for the future implementation of FSO systems through hybrid DWDM (dense wavelength division multiplexing) multi-beam FSO configurations.

FSO represents a technology for wireless data transmission, based on the optical propagation of signals through free space in the infrared part of the spectrum. For FSO systems, it is necessary to have LOS (Line Of Sight) for signal propagation. Some of the advantages of FSO systems include:

- compatibility of components with optical fibers,
- wide bandwidth that enables fast data transmission,
- flexibility allowing construction in inaccessible locations,
- unlicensed frequency range,
- Optical beam directionality providing high energy efficiency,
- Invisible optical beams are narrow and inconspicuous, making it difficult to detect their existence, which enhances data transmission security.

In addition to their advantages, there are also drawbacks that may limit their usage. The main drawbacks include limited range (line of sight) and susceptibility to signal attenuation due to atmospheric turbulence. Atmospheric turbulence is the most common limiting factor in the implementation process of FSO systems, occurring as a result of the greater heating of the air at Earth's surface compared to higher altitudes. This layer of warmer air, being less dense, rises and mixes turbulently with cooler air, causing random fluctuations in air temperature. The variations induced by turbulence can be observed as discrete cells or eddies with different temperatures, acting like refractive prisms with varying sizes and angles of refraction. Looking through the perspective of geometric optics, to consider these eddies as lenses that randomly refract light rays further results in creating distortions in the light beam at the signal receiver. Random changes in amplitudes due to interaction with eddies are referred to as scintillation. Signal attenuation caused by atmospheric turbulence can lead to errors in information transmission, reflected in incorrectly transmitted bits (BER - Bit Error Rate). To anticipate such conditions, it is necessary to perform statistical modeling of atmospheric turbulence. Due to the characteristics of atmospheric turbulence, there is no possibility of universally modeling atmospheric turbulence. In the paper [4], the characteristics of FSO systems were presented using the Gamma-Gamma model. The simulation results demonstrate good performance across various atmospheric turbulence intensity. This distribution has found application under conditions of low scattering and high refraction. In the paper [5], the exponential Weibull distribution is introduced, providing a good match for simulations and experimental research under all aperture averaging under weak or moderate turbulence.

The research involved the analysis of three different statistical models: gamma-gamma, Rician, and Nakagami-m. The results from this analysis are crucial as they can significantly facilitate the selection of an appropriate distribution and simplify the process of implementation of FSO systems. This research becomes particularly relevant when considering that designing wireless optical systems often requires combining various approaches. Additionally, designing a system is very often based on an analysis that combines the mentioned models.

2. GAMMA-GAMMA DISTRIBUTION

Gamma-Gamma model is based on the process of modulating fluctuations of the optical signal passing through a turbulent atmosphere, which has a low-scattering and high-refraction effect. This model describes atmospheric fluctuations of both large and small scales eddies, allowing for a broad range of applicability in atmospheric turbulence. According to the scintillation theory, the normalized received radiation X is defined as the product of two independent random processes X_x i X_y [4], [6] , [7]:

$$X = X_x \cdot X_y \tag{1}$$

$$f = \left(X_x\right) = \frac{a\left(aX_x\right)^{a-1}}{\Gamma(a)} e^{-aX_x}; X_x > 0; a > 0 \tag{2}$$

$$f = \left(X_y\right) = \frac{\beta\left(\beta X_y\right)^{\beta-1}}{\Gamma(\beta)} e^{-\beta X_y}; X_y > 0; \beta > 0 \tag{3}$$

If expression (1) is written as X_y=X/X_x and substituted into equation (3), the modified expression is obtained:

$$f\left(X / X_x\right) = \frac{\beta\left(\beta\frac{X}{X_x}\right)^{\beta-1}}{X_x\Gamma(\beta)} e^{-\beta\frac{X}{X_x}} \tag{4}$$

To obtain the unconditional irradiance distribution, the average of equation (4) using the gamma distribution from equation (2) is formed, resulting in the PDF of the new gamma-gamma distribution:

$$\begin{gathered} p_x(X) = \int_0^\infty f\left(X / X_x\right) f\left(X_x\right) dX_x = \\ \frac{2(a\beta)^{\frac{a+\beta}{2}}}{\Gamma(a)\Gamma(\beta)} X^{\frac{a+\beta}{2}-1} K_{a-\beta}\left(2\sqrt{a\beta X}\right),\ X > 0 \end{gathered} \tag{5}$$

where X is the signal intensity, Γ(.) gamma function, K(.) - modified Bessel function of the second kind of the *n*-th order. The parameters *α* and *β* are key factors of this distribution, signifying the presence of both large and small-scale scintillations.

$$a = \left[\exp\left(\frac{0.49\sigma_R^2}{\left(1+1.1\sigma_R^{\frac{12}{5}}\right)^{\frac{5}{6}}}\right) - 1\right]^{-1} \tag{6}$$

$$\beta = \left[\exp\left(\frac{0.51\sigma_R^2}{\left(1+0.69\sigma_R^{\frac{12}{5}}\right)^{\frac{5}{6}}}\right) - 1\right]^{-1} \tag{7}$$

where σ_R^2=1.23C_n^2 $k^{7/6}$ $L^{11/6}$ represents the Rytov variance and is used as a metric for the strength of atmospheric turbulence, classifying it as follows:

- 0 < $\sigma_R^2 \leq$ 0.3 - denotes weak turbulence,
- 0.3 < $\sigma_R^2 \leq$ 5 - denotes moderate turbulence,
- σ_R^2 > 5 - denotes strong turbulence.

The parameter C_n^2 represent the refractive index and is used as a measure of turbulence intensity, serving as a key element in turbulence classification. For horizontal signal propagation, C_n^2 ranges from 10^{-17} to 10^{-13} $m^{2/3}$, where an increase in the refractive index value is associated with an amplification of turbulence intensity effects. The parameter *k* represents the wave number and can be determined from the expression *k*=2*π*/*λ*, where *λ* represents the wavelength and determines the degree of fluctuation in the phase aspect of the signal. Longer wavelengths are characterized by less dispersion and reflection, which can reduce oscillations in the phase aspect of the signal and decrease the variance value. *L* represents the length of signal propagation, where longer distances directly influence the increase in the value of the Rytov variance.

Table 1. Parameter values under turbulence.

Parameter	Weak turbulence	Moderate turbulence	Strong turbulence
α	11.6	4	4.2
β	10.1	1.9	1.4
σ_R^2	0.2	1.6	3.5

Table 1 provides values of parameters under weak, moderate, and strong turbulence.

This model fits well with experimental radiation measurements. By applying the formula for the gamma-gamma distribution, various distribution shapes can be represented depending on the turbulence level, ranging from weaker, through moderate, to strong turbulence. The presented graph clearly shows the difference in the distribution depending on turbulence (Figure 1). It can be noted that as turbulence gets stronger, it leads to the broadening of the distribution, accompanied by an increase in the radiation range.

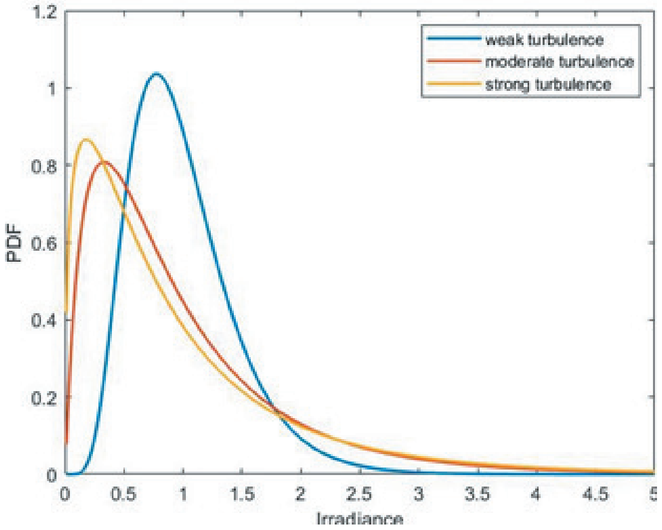

Figure 1. Gamma-gamma distribution under weak, moderate, and strong turbulence.

3. RICIAN DISTRIBUTION

Rician model is used to describe signal attenuation when there is one strong component corresponding to the line of sight between the transmitter and receiver, and much weaker components. This distribution model is used to describe terrestrial mobile channels in urban and sparsely populated areas, as well as in mobile satellite channels. The PDF of Rician fading can be represented using the expression:

$$p_x(x) = \frac{x}{\sigma_x^2} e^{-\frac{\left(x^2+s^2\right)}{2\sigma_x^2}} I_0\left(\frac{xs}{\sigma_x^2}\right),\ x > 0 \tag{8}$$

where I0(.) is the modified Bessel function of the first kind of zeroth order, σ_x^2 the power of scattered components, x represents the envelope of the desired signal, and s is the amplitude of the dominant component [6], [8], [9].

$$K = \frac{s^2}{2\sigma_x^2} \tag{9}$$

$$\Omega = s^2 + 2\sigma_x^2 \tag{10}$$

If the values of K and Ω are substituted in the previous expression, this yields the derived expression for the PDF of the Rician model:

$$p_x(x) = \frac{2x(K+1)}{\Omega} e^{-K-\frac{(K+1)x^2}{\Omega}} I_0\left(2x\sqrt{\frac{K(K+1)}{\Omega}}\right) \tag{11}$$

As seen from the expression, the parameters involved in the equation are the K-factor (Rician factor), signal envelope x and power factor Ω. The K-factor represents the ratio of components arising from the direct propagation of the signal along the line of sight (dominant component) and components resulting from scattering. The Rician factor takes values from 0, where the Rician model reduces to Rayleigh, to ∞ when there is no fading in the channel. Increasing the K factor contributes to better signal transmission characteristics. From the perspective of FSO systems, to improve the K-factor, it is necessary to increase the diameter of the receiver and transmitter, increase power and bandwidth. The signal envelope x varies according to the transmitted information, while the power factor Ω represents the total power of both components and acts as a scaling factor for the distribution.

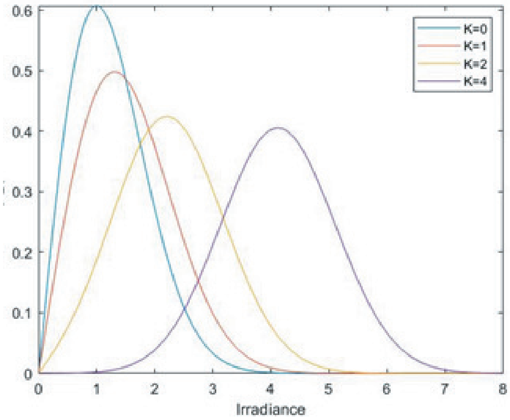

Figure 2. Rician distribution as a function of the K factor.

4. NAKAGAMI-M DISTRIBUTION

The Nakagami-m distribution depicts the signal envelope in channels with multiple clusters, where there is no dominant component, and the transmission is based on scattering components. The PDF of Nakagami-m fading can be represented using the expression [10], [11]:

$$f_V(\nu) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m V^{2m-1} e^{-\frac{m}{\Omega}V^2},\ z > 0,\ \mathrm{m} \geq \frac{1}{2} \tag{12}$$

The received signal level V represents the average signal strength at the receiver. A higher level indicates better conditions for signal reception, resulting in lower reception errors. The average signal power $\Omega=E[V^2]$ represents the mean power of the signal propagating through the channel. Higher power enables better communication channel characteristics and reduces the probability of reception errors. The fading distribution parameter m represents a crucial factor in the shape of the distribution. Higher values of the parameter m indicate a lower level of signal fluctuation, resulting in a lower level of fading. This parameter aids in adapting the distribution to different environments.

Parameter of the fading distribution is always $m \geq 0.5$. With an increase in this parameter, the system performance improves, and accordingly, some characteristics of this distribution can be defined:

- For the case when $m=1$ and $\Omega=2\sigma^2$, the result is the Rayleigh distribution as a special case (figure 4).
- For $m=0.5$ the distribution reduces to a one-sided Gaussian distribution (Figure 3).
- For $m \rightarrow \infty$, a model is created where there is no fading in the channel.

The figure illustrates an example of the Nakagami-m distribution where the parameter value is $m=0.5$ and $\Omega=1$. From the figure, it can be observed that the distribution has the shape of a one-sided Gaussian distribution.

In this example, the shape of the Nakagami-m distribution is illustrated as the parameter m increases. Values of $m=1$, $m=2$ and $m=3$ are taken as examples, while $\Omega=1$ remains unchanged. For the value $m=1$ the result is the Rayleigh distribution, as mentioned in its characteristics. As the values increase, the distribution takes on the shape of the Rician distribution with a larger amplitude of oscillation. This model provides the flexibility to adapt and model in various environmental conditions. In practice, measurements need to be conducted under diverse atmospheric conditions to obtain the parameters necessary for calculating mathematical expectations.

After analyzing the characteristics of the presented distributions, a table 2 has been constructed to provide a simplified overview of the key parameters of each distribution.

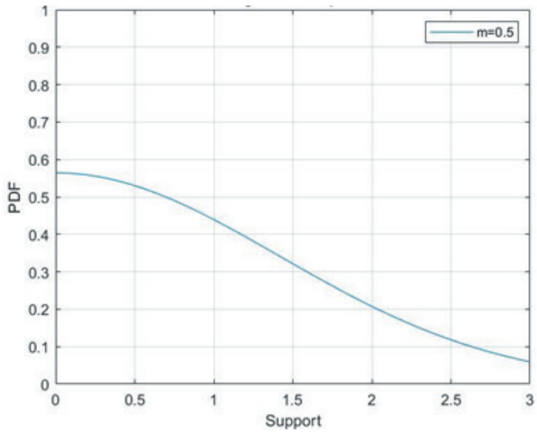

Figure 3. Nakagami-m distribution for the parameter value m=0.5.

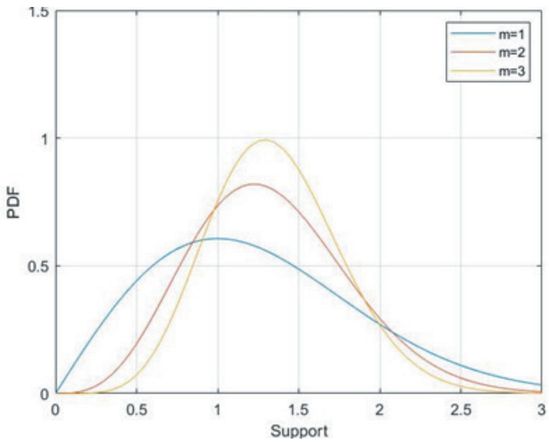

Figure 4. Nakagami-m distribution for parameter values m=1, m=2 and m=3.

Table 2. Key parameters of each distribution.

	Parameters	**Designation**	**Value description**		
	Atmospheric turbulence		Weak	Moderate	Strong
Gamma Gamma distribution	Refractive index Rytov variance	$C_n^2[m^{-2/3}]$ σ_R^2	10^{-17}-10^{-15} $0<\sigma_R^2\leq 0.3$	10^{-14} $0.3<\sigma_R^2\leq 5$	10^{-13} $\sigma_R^2>5$
	Wavelength	$\lambda[\eta m]$	Longer wavelengths lead to a decrease in the variance value, reducing signal variability.		
	Distance between transmitter and receiver	$L[m]$	Longer distance results in an increase in the value of the Rytov variance.		
Rician distribution	K-factor – ratio between dominant components and components resulting from scattering	K[dB]	For K=0 the Rician model reduces to Rayleigh Increasing the K factor contributes to better signal transmission characteristics, for K→∞ there is no fading in the channel.		
	Total signal power	Ω	Acts as a scaling factor for the distribution.		
	Signal envelope	x	Varies according to the transmitted information.		
Nakagami-m distribution	Parameter of the fading distribution	m	Parameter of the fading distribution is always m≥0.5. For the value m=1 the result is the Rayleigh distribution. Higher values of the parameter m indicate a lower level of signal fluctuation, For m→∞ there is no fading in the channel.		
	Received signal level	V	Higher level indicates better conditions for signal reception.		
	The average signal power	Ω	Higher power enables better communication channel characteristics and reduces the probability of reception errors.		

5. CONCLUSION

This paper introduces distributions used to describe signal attenuation in wireless communication systems. Based on the presented information, it can be concluded that there is no single model that would apply to all turbulent channels. Each distribution has its unique characteristics, and its applicability is limited to specific conditions and environments. For the Gamma-Gamma distribution, the principle of aggregating radiation is presented as the product of two independent random processes, each characterized by a gamma PDF. To adapt the system to atmospheric conditions, similar to how it was done with the Gamma-Gamma distribution, by integrating two different models, new combined models can be created. By creating such statistical models, it can significantly contribute to easier FSO system design, as each model can be adapted to atmospheric conditions. Therefore, it is crucial to have a profound understanding of the environment through which the signal will propagate to properly choose or create a statistical model.

6. REFERENCES

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