ADVANCED TECHNOLOGIES AND APPLICATIONS SESSION

APPLICATION OF THE 3D GEOGEBRA CALCULATOR FOR TEACHING AND LEARNING STEREOMETRY

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Abstract:

In this paper, we demonstrate how the 3D geometrical models can be generated using the dynamic mathematical software GeoGebra, and how can these models be used for teaching and learning Geometry, that is, stereometry.

Three geometrical problems are presented, processed using the 3D GeoGebra tools, and their features and possible applications are discussed. Special attention is paid to the ways to manipulate the presented materials, and also to the different possibilities for displaying and observing the objects generated in this way.

All the GeoGebra materials prepared for this paper can switch to a special mode for working with 3D glasses, and one section of the paper is dedicated to this GeoGeobra feature and the possibility of creating a real 3D environment.

The benefits of the application of these 3D models are discussed, and future remarks are given, considering the possible ways for using the 3D GeoGebra Calculator.

Keywords:

3D GeoGebra Calculator, Geometry, 3D Models, Stereometry, 3D Glasses.

INTRODUCTION

Geometry is one of the fundamental fields of mathematics. Mastering geometry knowledge and skills has always been a challenge for students. Imagining objects, especially in 3D space, establishing relationships between them, and solving geometric problems can sometimes be very demanding because they require simultaneous work with multiple representations, geometric, graphic, and algebraic.

When considering the process of learning geometry, the van Hiele Model of geometric thinking is mostly used. This model includes five levels through which students pass during the learning process, in order to gradually acquire knowledge and understanding of geometric concepts [1]. Taking this into account, the different approaches to teaching and learning geometry were explored, mostly the manipulative approach and the technology-based one [2]. It was confirmed that the technology-based approach had the best effects on students' achievements.

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Over the years, many different software were developed for the purpose of teaching and learning mathematics, especially geometry. The GeoGebra software and its tools for 2D, and later for 3D geometry and augmented reality stood out as one of the best for educational purposes. There are many researches and reports about the successful implementation of GeoGebra and its 3D geometry and augmented reality modules [3], [4], [5].

Concerning all the results from previous research, we have chosen GeoGebra software for developing materials in 3D and for 3D glasses which were applied as support for teaching stereometry and for solving problems in this field.

2. CREATING GEOMETRICAL MODELS USING THE 3D GEOGEBRA CALCULATOR

GeoGebra is dynamic mathematical software, designed for application in mathematics and sciences. It is open-source software, easy to use, but powerful concerning the possibilities of its tools.

GeoGebra has several modules, such as Graphing Calculator (for graphing functions, investigating equations, and plotting data), Geometry (for constructing circles, angles, transformations, etc.), CAS Calculator (for solving equations, expanding and factoring expressions, finding derivatives and integrals), and 3D Calculator (for graphing 3D functions, plotting surfaces and doing 3D geometry). All modules are available for use online or download, and all of them can work on all platforms (Windows, IOS, Linux, Android).

3D Calculator is especially interesting for application when teaching and learning stereometry. It is equipped with multiple tools, such as the tool for creating planes (using three points, perpendicular or parallel plane), intersecting surfaces, drawing prisms, cones, cubes, spheres, etc. All the tools mentioned above are located in the upper left corner of the 3D Calculator window, Figure 1.

In the upper right corner of the window, there are more tools, for choosing the type of projection, rotation, etc., Figure 1.

We used the GeoGebra 3D Calculator for creating materials that can be applied as additional help for solving problems in stereometry. For each problem, we have made 3D model, available for public use on GeoGebra's official website.

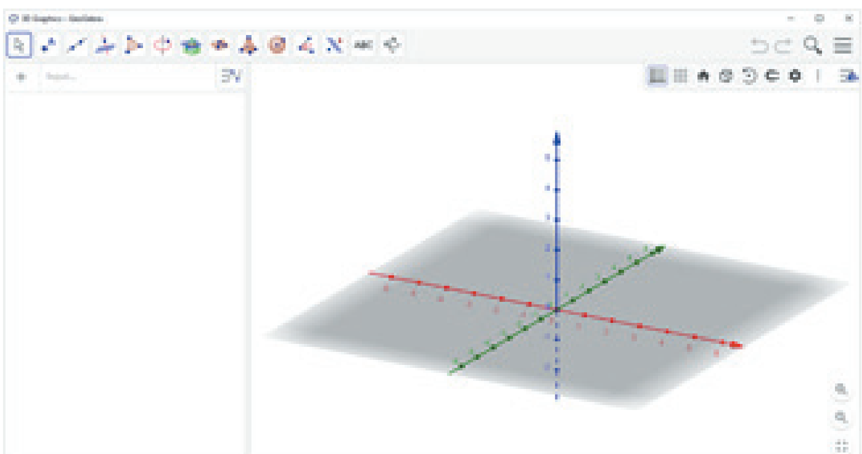

Figure 1 – The window of the GeoGebra 3D Calculator.

Problem 1:

A hexahedron $ABCDA_1B_1C_1D$ of edges a is given. If point S is the middle point of the edge BB_1, determine the area of intersection of the prism and the plane containing the vertices A_1 and C_1 and the given point S.

The 3D GeoGebra model of the hexahedron from Problem 1 is represented in Figure 2.

Using this 3D representation the students can see the precise drawing of the geometrical object and therefore, they can gain better insight into the problem and its solution. But, a far more important and valuable is the possibility of a 3D GeoGebra Calculator to manipulate the 3D object, meaning, it can be rotated and observed from different points of view, Figure 3.

The rotation of the 3D object and observing it from different points of view can be easily achieved by simply "dragging" the object using a pointer. This feature of GeoGebra, especially the 3D Calculator is the greatest possible help students can get when learning stereometry. By carefully examining the difficulties students face in geometry, it is determined that the biggest problem is the impossibility of imagining the space and presenting (sketching) it in the best way in order to understand the problem. The material for Problem 1 is available online, on the official GeoGebra website [6].

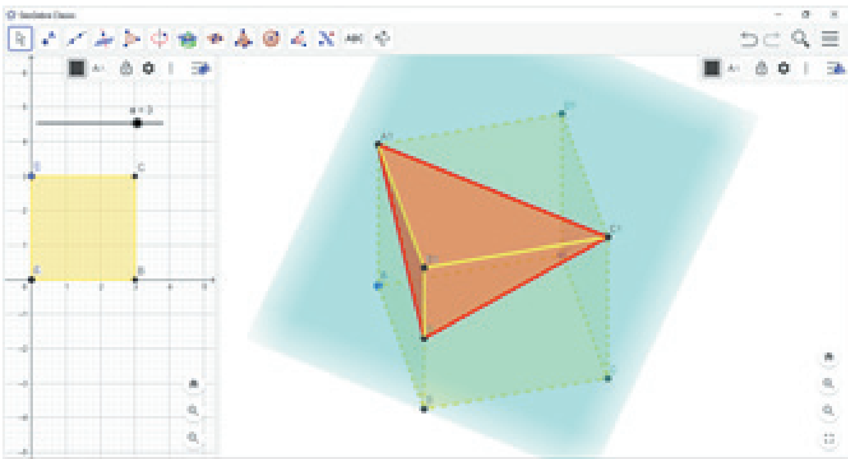

Figure 2 – 3D Representation of Problem 1.

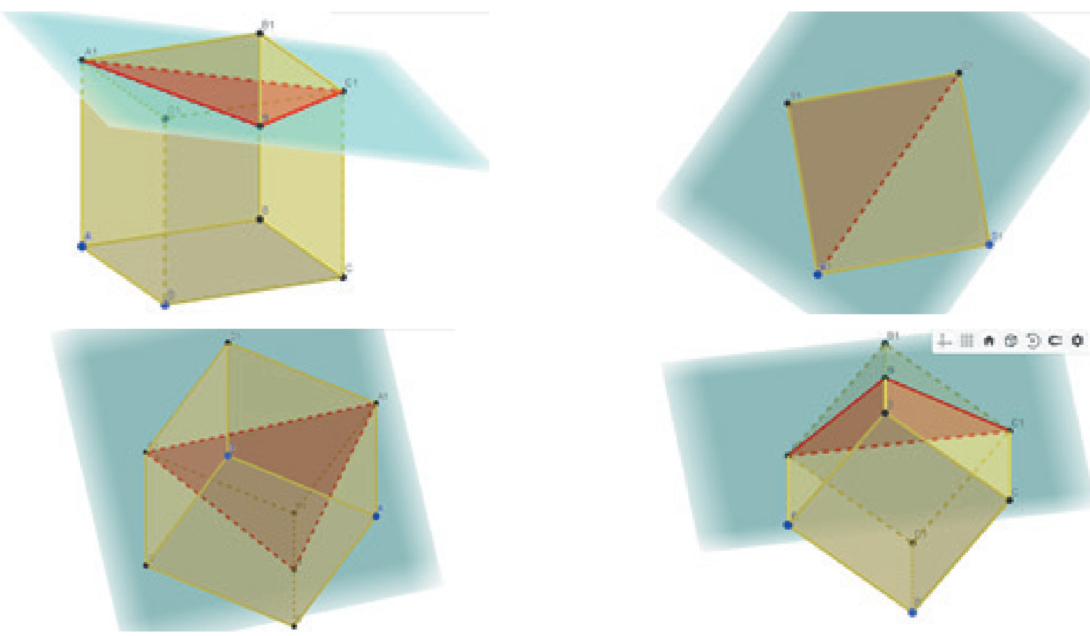

Figure 3 – Different points of view for Problem 1.

Problem 2:

A regular three-sided equilateral prism $ABCA_1B_1C_1$, edge a is given. If point E is the middle point of the edge A_1C_1, and point F is the middle point of the edge B_1C_1, determine the cross-sectional area of the prism and the plane containing the segment EF and the prisms' edge AB.

As for the previous problem, the 3D Calculator was used for representing a 3D object from Problem 2. Some of the different views of the object are presented in Figure 4.

The material for Problem 2 is also available online, on the official GeoGebra website [7].

Problem 3:

Determine the intersection of a regular four-sided prism and a plane parallel to its base.

For the purpose of this problem, we used the GeoGebra tool called *Slider*, in order to enable additional "movement" of the intersection plane, Figure 5.

By moving the slider *b* from the upper left corner of the GeoGebra window, the plane will move parallel to the base of the prism, allowing students to see its different positions, Figure 6.

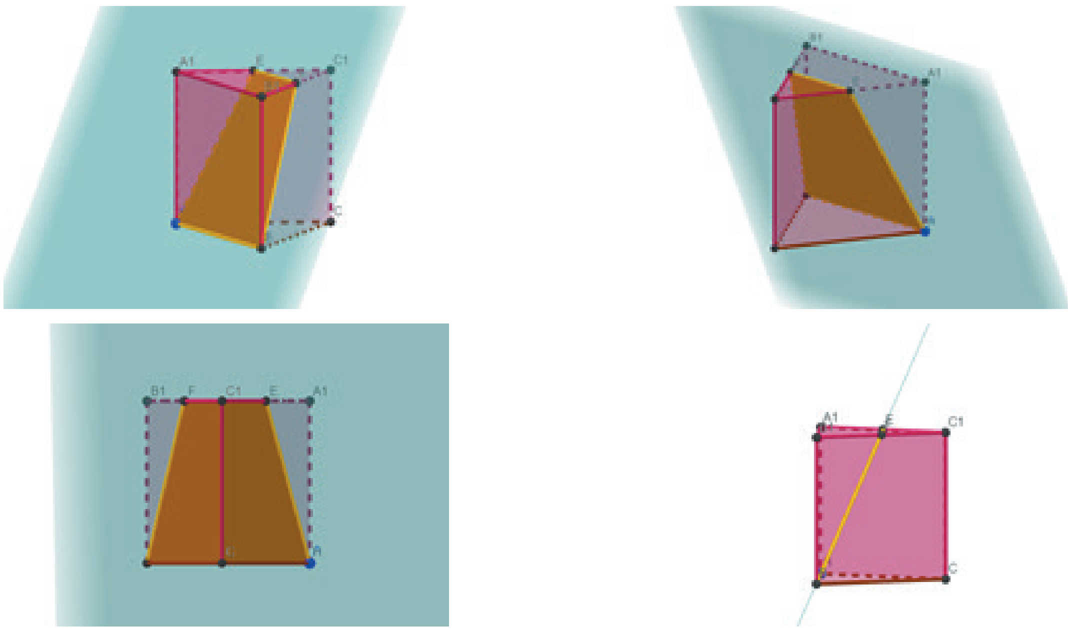

Figure 4 - Different points of view for Problem 2.

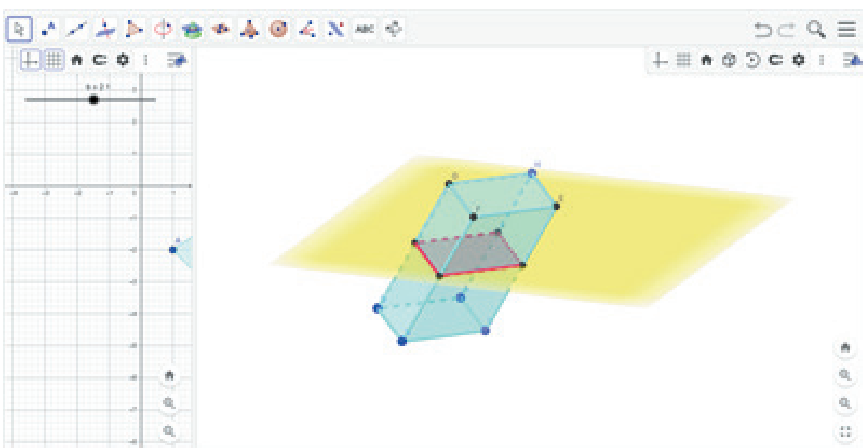

Figure 5 – Intersection of the prism and plane for Problem 3.

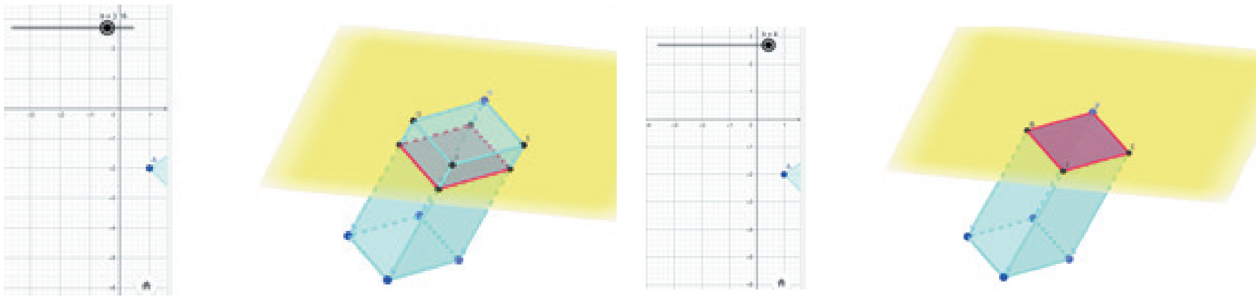

Figure 6 – Different positions of the plane for Problem 3.

The material for Problem 3 is available on the official GeoGebra website [8].

From the problems presented above it can be seen that the 3D representation of the geometrical objects is easy to create by using the 3D GeoGebra Calculator. The benefits of such materials can be various, and one of the important ones is the possibility for students to "see" the 3D space to manipulate it.

3. THE GEOGEBRA MODE FOR WORKING WITH 3D GLASSES

GeoGebra 3D Calculator has special tools for transforming 3D drawings into those adapted for the application of specialized 3D glasses. The tools for choosing a projection for 3D glasses are positioned in the upper right corner of the GeoGebra window, Figure 7.

By using the tool where the glasses are marked, GeoGebra automatically transforms the existing drawing into the one adapted for 3D glasses, Figure 8.

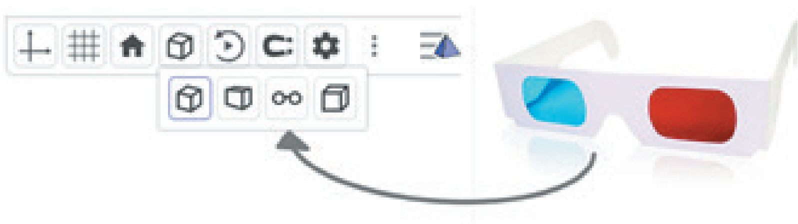

Figure 7 – GeoGebra tools for choosing projection type and 3D glasses.

a) b)

Figure 8 – Transformation of 3D object a) into an adapted object for 3D glasses b).

When using the 3D glasses on objects transformed in this way, the insight into "depth" is enabled and a complete sense of space is achieved. The possibility of rotating the objects and observing them from different points of view is also available. The tool for adapting 3D objects for 3D glasses is applicable to all objects created using GeoGebra 3D Calculator. All problems presented in this paper and their corresponding materials on GeoGebra's official website can be adapted for 3D glasses by using the above-mentioned tool.

4. DISCUSSION AND FUTURE REMARKS

The materials presented in the previous sections of the paper can be used for teaching geometry (stereometry) at all levels of mathematics education. The benefits of this way of representing geometric objects are multiple.

First, using GeoGebra, it is possible to be much more precise, which contributes to a simpler insight into the possible solutions to the geometric problems. The possibility for multiple representations (algebraic, numerical, graphical, dynamical) that GeoGebra offers especially contributes to analyzing and solving problems.

The 3D Calculator tools for creating 3D objects are immeasurable and help students who have a problem with imagining space and objects in it. The possibility to manipulate objects and observe them from different points of view can induce a higher level of understanding of space and relations within it.

Adaptation of 3D objects for using 3D glasses is a particularly good opportunity that GeoGebra provides. It is possible to observe objects in real 3D space, including depth. Very important to mention is that in this case there are no major requirements regarding technical equipment, GeoGebra is open-source software and glasses are available to everyone for their symbolic value.

Future remarks include further upgrades and application of GeoGebra materials developed in the 3D Calculator. It is important to note that GeoGebra has a module for augmented reality which is more sophisticated and offers even more possibilities for working in 3D surroundings.

5. CONCLUSION

Technology and modern software tools have proven themselves to be of great help to teachers and students to enable a better understanding of mathematical concepts. This is especially important concerning teaching geometry and stereometry where students have to develop geometric thinking and the skills to imagine the 3D space and relations between objects within it.

GeoGebra software and its modules and tools proved to be very effective for educational purposes. The 3D modules for creating 3D objects have more than enough various tools for drawing 3D objects and possibilities for their display and manipulation.

Also, the option for adaptation of 3D GeoGebra materials for the use of 3D glasses adds one more useful opportunity for creating an environment that can contribute to deeper understanding and better visualizing of 3D space.

Further application of 3D GeoGebra modules goes toward the development of more sophisticated 3D materials combined with 3D glasses and the use of augmented reality.

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