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APPLICATION OF THE MS EXCEL ON NUMERICAL SOLVING OF ORDINARY DIFFERENTIAL EQUATIONS

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Abstract:

In classical physics, and especially in engineering, a large number of natural phenomena are described by ordinary differential equations. The majority of those differential equations do not have an analytical solution, which implies that In those situations, the solution must be sought numerically. That's when MS EXCEL can be applied.

In this paper, we will show and explain one method for the application of MS EXCEL on solving and creating graphical representations of their solutions.

Also, the obtained numerical solution of differential equations from MS EXCEL will be compared with their analytical solution.

Keywords:

Differential equations, MS EXCEL, Analytical solutions, Numerical solutions, Graphical representation.

INTRODUCTION

Many natural phenomena, the description of which deals with classical physics, result in mathematical terms, namely, algebraic and differential equations. These equations, primarily of a linear character, often have an analytical solution, while those equations that are of a non-linear character, usually do not have an analytical solution. For solving them, only numerical analysis that uses various types of successive algorithms remains available. The application of these algorithms is very complicated and requires special computer programs and applications.

The program package MS EXCEL is proven to be very effective for solving differential equations, although it has been noted that it is not so often used by engineers. However, the advantages of MS EXCEL are various, starting from availability to the wide range of users, and simplicity of its use but still software that can solve very complicated problems.

In this paper, based on the comparative analysis of the analytical and numerical solutions of the differential equation, we will present how MS EXCEL can be used in engineering practice for more effective solving of differential equations and for graphical representation of their solutions.

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Comparative Analysis of Analytical and Numerical Solutions of Differential Equations

In the further course of this text, the analytical and numerical solution of the linear homogeneous differential equation of the first order with constant coefficients will be considered. This type of equation is easily solved analytically. It is clear that if the corresponding equation has an analytical solution, there is no need to look for a numerical solution. However, here, in addition to the analytical one, a numerical solution will be intentionally presented in order to perform a comparative analysis of the obtained results. Of vital importance here is the question of accuracy, that is, to determine to what extent the numerical solution deviates from the analytical one.

The mentioned differential equation has the following form:

$$\frac{dy(x)}{dx} + ay(x) = 0; a \neq 0.$$

Equation 1 – Differential equation.

Along with this differential equation follows the initial condition:

$$y(x)\big|_{x=x_0} = y(x_0) = y_0$$

Equation 2 – Initial condition for the differential equation.

It is easy to show that the analytical solution of this differential equation, with the mentioned initial condition, corresponds to the following expression (explained in [1]):

$$y(x) = y_0 \cdot e^{-a \cdot x}$$

Equation 3 – Differential equation used in this example.

Let us now take some concrete values for the above constants, for example:

$$a=2 \tag{1}$$

$$x_0=0 \tag{2}$$

$$y_0=3 \tag{3}$$

Equation 4 – Values for the constants.

With the constants defined in this way (1), (2) and (3), expressions from Equation 1, Equation 2, and Equation 3 take the following form:

$$\frac{dy(x)}{dx} + 2y(x) = 0, \tag{4}$$

$$y(x)\big|_{x=x_0=0} = y(x_0) = y(0) = y_0 = 3 \tag{5}$$

$$y(x) = 3 \cdot e^{-2 \cdot x} \tag{6}$$

Equation 5 – The form of differential equation and initial condition.

Now it is necessary to draw a graph of the analytical solution given by (6). We will look for the values of the function $y(x)$ in the following points:

$$x_0=0 \tag{7}$$

$$x_1=x_0+1 \cdot \Delta \tag{8}$$

$$x_2=x_0+2 \cdot \Delta \tag{9}$$

$$\dots$$

$$x_n=x_0+n \cdot \Delta \tag{10}$$

Equation 6 – Points for observing the value of the function y(x).

Let's take it, for example: $\Delta=0,1$ and $=(0,1,2,\dots,40)$. The points $x_0, x_1, x_2, \dots, x_n$ have the following values:

$$x_0=0 \tag{11}$$

$$x_1=0,1 \tag{12}$$

$$x_2=0,2 \tag{13}$$

$$\dots$$

$$x_{40}=4 \tag{14}$$

Equation 7 – The observed points concerning the adopted values for Δ and n.

The values of the function $y(x)$, given by (6) in selected points, (11), ... ,(14), are as follows:

$$y_0=y(x_0)=3 \tag{15}$$

$$y_1=y(x_1)=2,456192 \tag{16}$$

$$y_2=y(x_2)=2,010960 \tag{17}$$

$$\dots$$

$$y_n=y(x_{40})=0,001006 \tag{18}$$

Equation 8 - The values of the function $y(x)$ for selected points.

All previously presented data are shown in Table 1, and the graph of function $y(x)$ is presented in Figure 1.

Let us now look for the numerical solution of this problem given by (4) and (5).

The first derivative of the function $y(x)$ can be approximated by the following expression, obtained from [2]:

$$\frac{dy(x)}{dx} \approx \frac{y^*_{n+1} - y^*_n}{\Delta}$$

Equation 9 – Approximation of the function $y(x)$.

Substituting the previous expression (Equation 9) into (4), and taking into account that the function $y(x)$ itself now takes its values at selected points of the independent variable (x), it will follow:

$$\frac{y_{n+1}^* - y_n^*}{\Delta} + 2 \cdot y_n^* = 0 \qquad (19)$$

$$y_{n+1}^* - y_n^* + 2 \cdot \Delta \cdot y_n^* = 0 \qquad (20)$$

$$y_{n+1}^* - \left(1 - 2 \cdot \Delta\right) \cdot y_n^* = 0 \qquad (21)$$

$$y_{n+1}^* = \left(1 - 2 \cdot \Delta\right) \cdot y_n^* \qquad (22)$$

Equation 10 – Algorithm for the numerical solution of the differential equation.

Now we should use the algorithm derived above and calculate the approximate (approximated) values of the function $y^*(x)$ at the selected points (11), ... ,(14).. These values are:

$$y_0^* = 3 \qquad (23)$$

$$y_1^* = \left(1 - 2 \cdot \Delta\right) \cdot y_0^* = \left(1 - 2 \cdot 0,1\right) \cdot 3 = 2,4 \qquad (24)$$

$$y_2^* = \left(1 - 2 \cdot \Delta\right) \cdot y_1^* = \left(1 - 2 \cdot 0,1\right) \cdot 2,4 = 1,92 \qquad (25)$$

$$\ldots$$

$$y_{40}^* = \left(1 - 2 \cdot \Delta\right) \cdot y_{39}^* = \left(1 - 2 \cdot 0,1\right) \cdot 0,000498 = 0,000399 \qquad (26)$$

Equation 11 - The approximate (approximated) values of the function $y^*(x)$ at the selected points.

As last time, all previously presented data are shown in Table 1, and the graph of the function $y^*(x)$ is shown in Figure 1. Finally, it should be said that in each step the difference between the exact analytical and approximate numerical solution was calculated, using the following expression:

$$\Delta^* = y\left(x\right) - y^*\left(x\right)$$

Equation 12 - The difference between the exact analytical and approximate numerical solution.

Table 1- Analytical and numerical solution of differential equation with the initial condition, Equation 5.

| (x)	$y(x)$	$y^*(x)$	Δ^*
0	3	3	0
0,1	2,456192	2,4	0,056192
0,2	2,01096	1,92	0,09096
0,3	1,646435	1,536	0,110435
0,4	1,347987	1,2288	0,119187
0,5	1,103638	0,98304	0,120598
0,6	0,903583	0,786432	0,117151
0,7	0,739791	0,629146	0,110645
0,8	0,60569	0,503316	0,102373
0,9	0,495897	0,402653	0,093243
1	0.406006	0.322123	0.083883
1,1	0,332409	0,257698	0,074711
1,2	0,272154	0,206158	0,065995
1,3	0,222821	0,164927	0,057894
1,4	0,18243	0,131941	0,050489
1,5	0,149361	0,105553	0,043808
1,6	0,122287	0,084442	0,037844
1,7	0,10012	0,067554	0,032566
1,8	0,081971	0,054043	0,027928
1.9	0,067112	0,043235	0,023878
2	0,054947	0,034588	0,020359
2,1	0,044987	0,02767	0,017317
2,2	0,036832	0,022136	0,014696
2,3	0,030156	0,017709	0,012447
2,4	0,024689	0,014167	0,010522
2,5	0,020214	0,011334	0,00888
2,6	0,01655	0,009067	0,007483
2,7	0,01355	0,007254	0,006296
2,8	0,011094	0,005803	0,005291
2,9	0,009083	0,004642	0,00444
3	0,007436	0,003714	0,003722
3,1	0,006088	0,002971	0,003117
3,2	0,004985	0,002377	0,002608
3,3	0,004081	0,001901	0,00218
3,4	0,003341	0,001521	0,00182
3,5	0,002736	0,001217	0,001519
3,6	0,00224	0,000974	0,001266
3,7	0,001834	0,000779	0,001055
3,8	0,001501	0,000623	0,000878
3.9	0,001229	0,000498	0,000731
4	0,001006	0,000399	0,000608
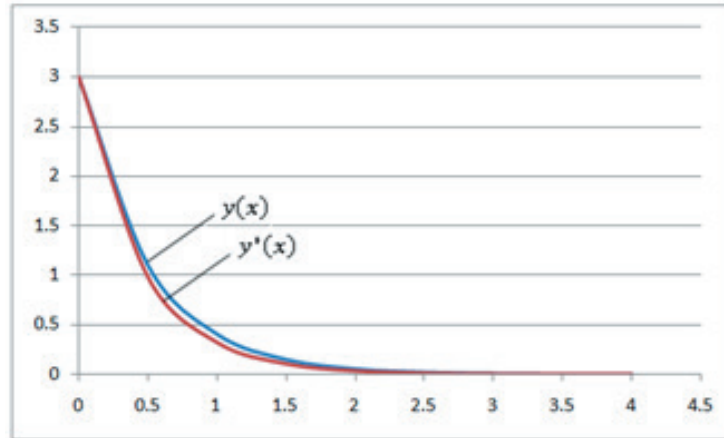
Figure 1 – Comparative graphs of analytical and numerical solutions of differential equation with the initial condition, (4), (5) and (6).
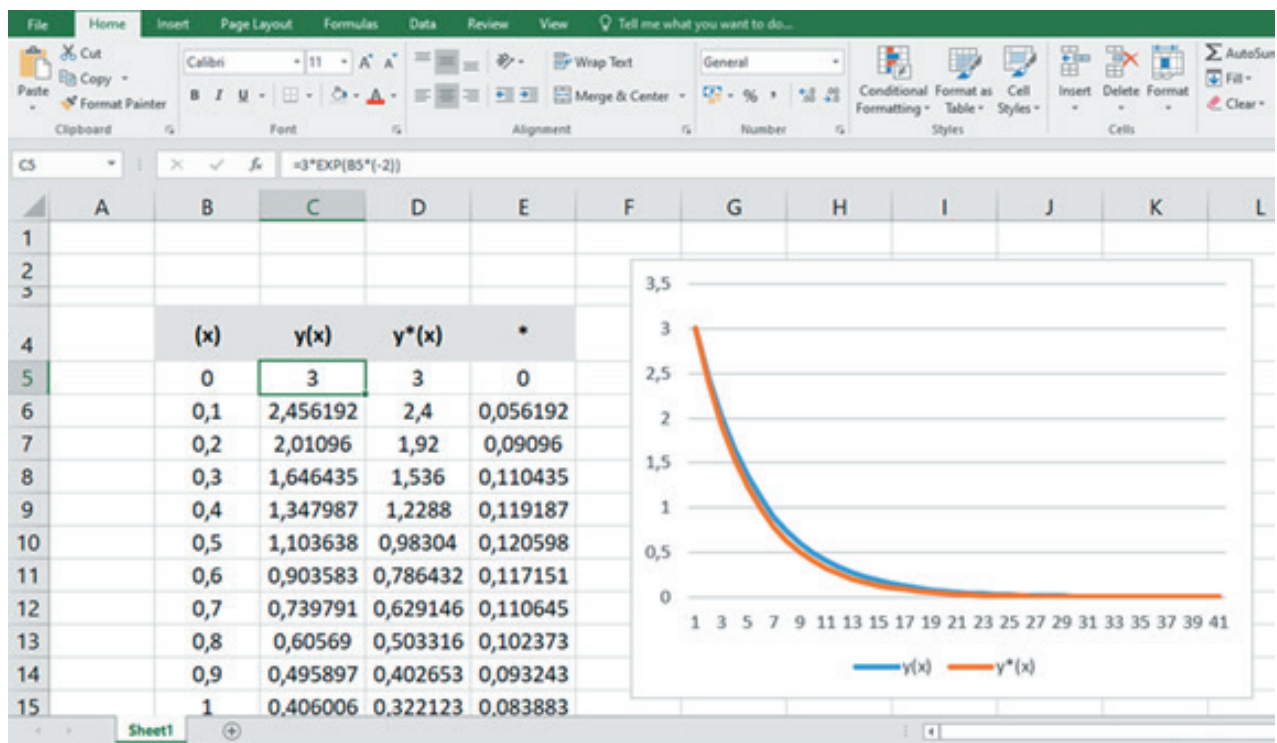

Figure 2 – Presentation of the solution of the differential equation using the MS EXCEL package.

All the obtained results, in this case, could be done by direct calculating, without using some additional software help. Nevertheless, taking into consideration that in engineering practice, there are problems described with very complicated differential equations, we wanted to present how MS EXCEL can be used for solving these kinds of problems.

All the necessary calculations, tabulation of data, and drawing of function graphs were done with the help of the MS EXCEL program package. In this simple example,

we can see that with the help of this program package, it is possible to perform various types of mathematical analysis very efficiently, easily, and comfortably, using functions and other features of MS EXCEL, Figure 2.

The great advantage of this package is that it is relatively cheap compared to competing packages and that it is available to a wide range of users.

3. DISCUSSION OF THE OBTAINED RESULTS AND FURTHER APPLICATION

In this chapter, two processes from technical practice will be briefly described, one is from the field of Fluid mechanics, while the other is from the field of Heat and mass exchange. Both processes are described by systems of differential equations and their solutions are sought with the help of numerical analysis. The complete processing of the results was carried out in the MS EXCEL program package. The first process considers the flow of a suspension (a two-phase mixture of solid and liquid phases) moving between two axial cylinders. The mathematical model of suspension flow describes the velocity field of the mixture as a whole (v) and the micro rotation velocity field of the solid phase within the mixture (ω), depending on the radial coordinate (r). This model is defined by a system of two ordinary linear differential equations of the second order with variable coefficients. An analytical solution to this problem is presented in reference [3], but it is extremely complicated and is given in the form of special functions. In order to confirm the accuracy of the analytical solution, a numerical solution was developed, which is presented in reference [4]. The comparative analysis of the analytical and numerical solution showed an extremely good agreement of the results (there is a negligibly small difference).

The second process considers the exchange of heat and mass between the drying agent - unsaturated moist air and moist materials of natural products. Namely, warm unsaturated moist air transfers heat to moist material, at the expense of which the moisture in it evaporates. In [5], the issue of drying natural materials in large industrial plants - belt dryers - was considered. Drying of a thick layer of wet material on a conveyor belt is described by a system of nonlinear partial differential equations [5], while a simplified variant of this problem - drying of a thin layer of wet material on a conveyor belt is described by a system of nonlinear ordinary differential equations [6]. The results of the numerical solution of these equations were compared with the results of measurements on a real process. The agreement of the results is quite good with a present error that is within the expected value.

Both problems, mentioned above, have very demanding solving procedures. In our practice, MS EXCEL was of great help, considering the ease of use and availability to students and engineers [7].

4. CONCLUSION

Based on the example of obtaining analytical and numerical solutions of differential equations, we have presented how can MS EXCEL program package be applied to solving this kind of problem, and how can it be used for graphical representation and analysis of the solutions.

Also, two examples were given that present real technical processes. These processes are described by complex mathematical models that require the application of numerical analysis. Application of some specialized software packages for this purpose in these cases gave almost no results. Unlike them, the MS EXCEL program package served the purpose perfectly and solved the mentioned problems very efficiently.

Interestingly, engineers do not use this package very often in practice, that is, they rarely use it for numerical analysis. Based on our experience, MS EXCEL should be used more in engineering practice, considering all its features.

5. REFERENCES

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