TEACHING TRIGONOMETRY ONLINE USING GEOGEBRA DYNAMICAL MATERIALS

Tanja Sekulić[1]*,
Goran Manigoda[2],
Valentina Kostić[3]

[1]Technical College of Applied Sciences in Zrenjanin, Zrenjanin, Serbia

[2]Gymnasium Zrenjanin, Zrenjanin, Serbia

[3]Academy of Applied Technical and Preschool Studies Niš, Department Pirot, Pirot, Serbia

Abstract:

In this paper, we present our experience in teaching mathematics online using GeoGebra as a software tool. The GeoGebra teaching materials developed and used for the online trigonometry course for Gymnasium students are shown and described in detail.

The experiences of teachers and students are presented and discussed. Also, in the paper are presented the research results related to the achievements of students during the online and classroom teaching and learning process.

The research results indicated that the application of GeoGebra materials during the online teaching process had a positive impact on maintaining the admissible level of students' achievements.

Keywords:

Trigonometry, GeoGebra, dynamical materials.

INTRODUCTION

Education worldwide is currently facing major challenges, considering the conditions imposed by the pandemic. Both teachers and students are faced with new ways of teaching and learning. With the help of technology, we have managed to sustain the educational process, but we encountered many obstacles. In the spring of 2020, when new conditions and restrictions emerged, it was necessary to establish, as soon as possible, an online teaching process. At the time, we did not have at our disposal the technical equipment needed for real-time online teaching (such as pen tables). The mathematics teaching process was particularly vulnerable because, to be able to give our students the maximum, it was necessary to provide them with learning conditions that would correspond to the direct contact we had in the classroom.

Trigonometry is one of the main subjects in the high school mathematics curriculum. Understanding trigonometry is very demanding for students even in normal conditions because they need to adopt the three representations at the time: algebraic, geometric, and graphic [1].

Correspondence:
Tanja Sekulić

e-mail:
tsekulicvts@gmail.com

DOI: 10.15308/Sinteza-2022-121-128

In an online classroom and without the possibility for real-time contact, we had to find a way to show our students all three representations. We have chosen to use GeoGebra software as a tool for an easier understanding of basic trigonometric concepts and functions. The GeoGebra software was chosen because it provides the possibility of multiple representations, and also for its dynamic nature suitable for making simulations and animations.

2. GEOGEBRA SOFTWARE

GeoGebra is open-source dynamic mathematical software that can be used to implement various mathematical theories. It is designed both, for scientific and educational purposes. One of the main GeoGebra's features is the possibility for multiple representations of concepts: algebraic, geometric, and graphic. In this way, objects can be displayed in multiple representations at the same time and it is possible to manipulate them. All representations are related and the change of parameters in one representation automatically implies changes in other representations. In that way, GeoGebra allows us to manipulate the objects and variables, that is, to make simulations and animations.

GeoGebra supports multiple platforms and can be installed on computers, tablets, and mobile phones. When installed, the user can create materials, but there is also the possibility of using ready-made materials available on the official GeoGebra site [2]. In that way, the materials can be used or shared at any time.

When it comes to its application for educational purposes, the main feature of GeoGebra is its ease of use and low requirements regarding the training of teachers and students for its application in teaching and learning practice. However, this is very powerful software, with the ability to meet the requirements of all users. The dynamic nature of GeoGebra is one of the main reasons for its use in STEM education [3]. Due to its visual and dynamic properties, GeoGebra is very often mentioned in research in education and teaching [4]. Today, GeoGebra is a software that is constantly present at all levels and in many fields of education, especially in the teaching of mathematics and sciences [5], [6].

For all its features, we have chosen GeoGebra as a supporting tool for teaching trigonometry. There were reported positive experiences in teaching trigonometry using GeoGebra [7], [8]. Those positive experiences were related to student achievements, understanding of basic trigonometric concepts, and also to the motivation of students for learning.

3. GEOGEBRA ASSISTED TRIGONOMETRY LEARNING

In the spring of 2020, we started with the online teaching process. In Gymnasium Zrenjanin was organized using the Google Classroom platform. The existing teaching schedule was realized in such a way that the teachers sent students written materials that contained lessons that were processed according to the curriculum. The materials for mathematics contained all the concepts and explanations related to the topic being covered, tasks and detailed solving procedures, as well as practice tasks. Students studied these materials and were able to ask the teacher a question using the Google Classroom option for comments. But, this kind of communication was slow and the lack of direct communication was more than obvious. However, the students were able to follow the materials at the beginning while topics started at school were still being addressed. When trigonometry had to be started, there were some concerns present about how will students learn and understand some completely new concepts to them using only written materials.

For that purpose, we decided to use technology to enrich the written materials with the GeoGebra dynamic elements which could help students to see and to better understand the connections between trigonometry concepts by giving them the possibility to manipulate the objects within those dynamic materials. For teaching trigonometry online, we used GeoGebra materials that we previously have made and designed. We uploaded those materials as resources on the official GeoGebra site. We made five GeoGebra materials, considering all the topics covered by the trigonometry course. Those topics were as follows:

1. Unit circle;
2. Reduction to the first quadrant;
3. Graphs of trigonometric functions;
4. Trigonometric functions with parameters;
5. Trigonometric equations.

For each of these topics, the link toward the matching GeoGebra material was included in the written material. The link provided students with access to the GeoGebra material which they could use for learning and practice.

3.1. UNIT CIRCLE

The unit circle is the basic concept with which students first get acquainted when learning trigonometry. They have to master and understand the representation of four trigonometric functions (*sinx, cosx, tgx* and *ctgx*). For that purpose, we created GeoGebra material as presented in Figure 1.

In the left window, there is a graphic representation of the unit circle. All four trigonometric functions are labeled and marked in different colors. On the circle, there is a point *M* that can be "dragged" on the circle thus changing the angle α. In that way, students can observe how a change in angle affects the change of the values and signs of trigonometric functions. The values and signs of the trigonometric functions are displayed in the right window, and the change of position of the point *M* on the unit circle corresponds to the change of values in the right window.

3.2. REDUCTION TO THE FIRST QUADRANT

With the reduction to the first quadrant students usually have a lot of problems, mostly due to misconceptions about unit circle and signs of trigonometric functions. To avoid that, we created GeoGebra material, which also consists of two windows, Figure 2. The window on the left is used for geometric representation. The students can move the point *M* on the circle, changing the angle φ that is, the quadrant in which the angle φ is located. At the same time, on the circle are represented the angle φ (marked yellow), and the two other angles used for reduction to the first quadrant, angle α (marked blue, used when reduction is realized as, $\varphi = a \pm k\pi, k \in Z$ and angle β (marked red, used when reduction is realized as $\varphi = a \pm k\frac{\pi}{2}, k \in Z$. In the window on the right, there is an algebraic representation of reduction to the first quadrant, using both angles, α and β.

In this material, the three representations (graphic, algebraic and dynamic) provided students with a deeper understanding of the problem of reduction to the first quadrant because they could experiment with angles in all four quadrants and at the same time observe the connections between the angles involved in reduction.

3.3. GRAPHS OF TRIGONOMETRIC FUNCTIONS

The GeoGebra material for trigonometric functions is presented in Figure 3.

In the left window, there are four checkboxes, one for each trigonometric function and the unit circle. By checking one function in the left window students can observe its graph in the right window. Both windows are dynamically connected, meaning, when students move the point *M* on the circle in the left window, the function is plotted in the right window.

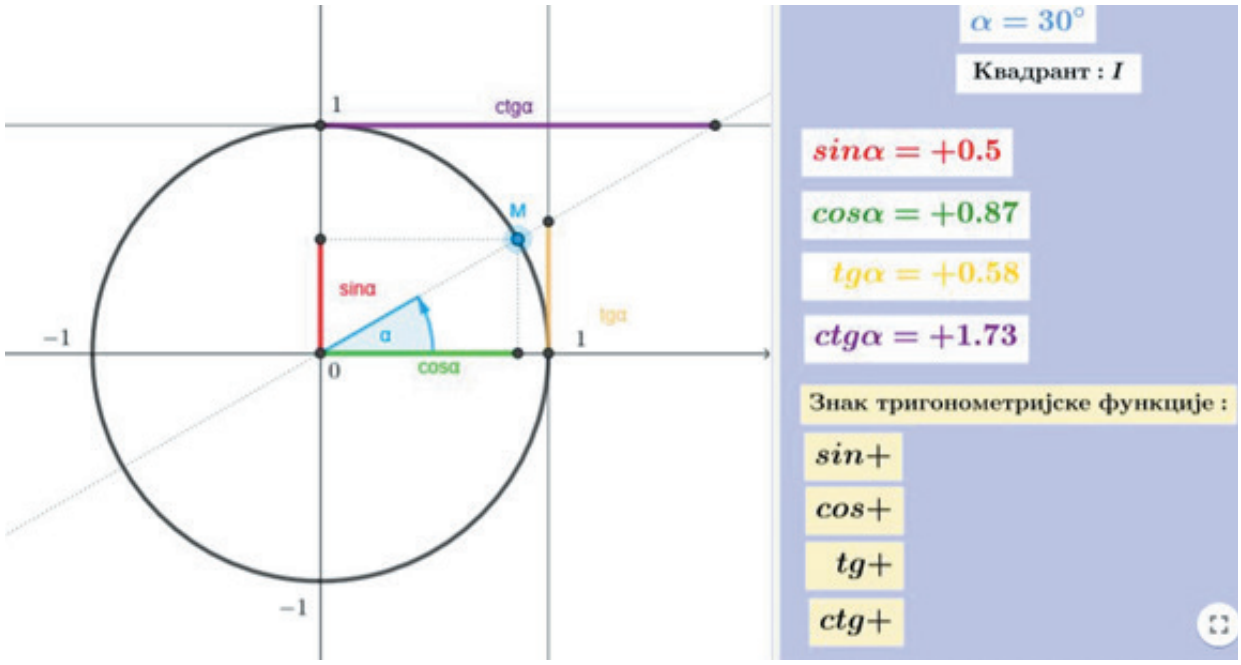

Figure 1 – Unit circle.

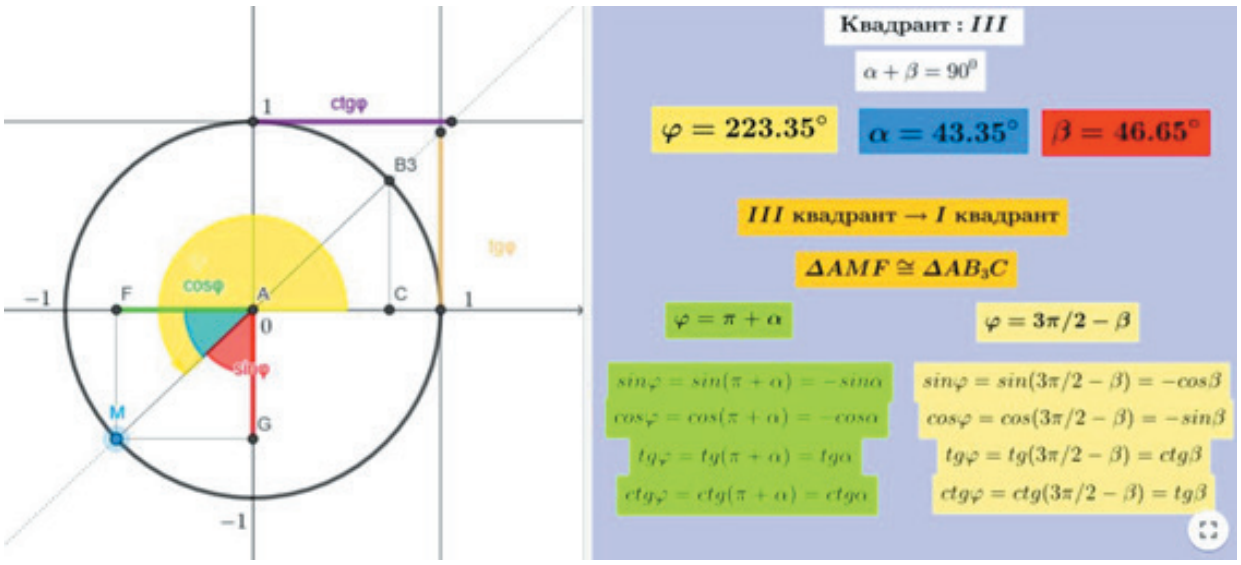

Figure 2 – Reduction to the first quadrant.

Also, the functions periods are highlighted in different colors. It is also possible to check more than one checkbox in the left window so that it can be observed more functions' graphs at the same time. We have included this possibility in the GeoGebra material so that students can see the similarities and differences between the graphs of trigonometric functions.

3.4. TRIGONOMETRIC FUNCTIONS WITH PARAMETERS

One GeoGebra material was made for trigonometric function with parameters, Figure 4. It is also designed with two windows. In the left windows, there are four checkboxes, for each trigonometric function shown in the form of three parameters: *y=a f(bx+c)*. Below checkboxes, there are three sliders, one for each parameter: *a, b, c.*

Students could change the parameters' values by moving the sliders and at the same time observe in the right window how the graph of a trigonometric function is changed (the graph is marked red) and also they could compare it to the graph of the basic trigonometric function without parameters (graph is marked black).

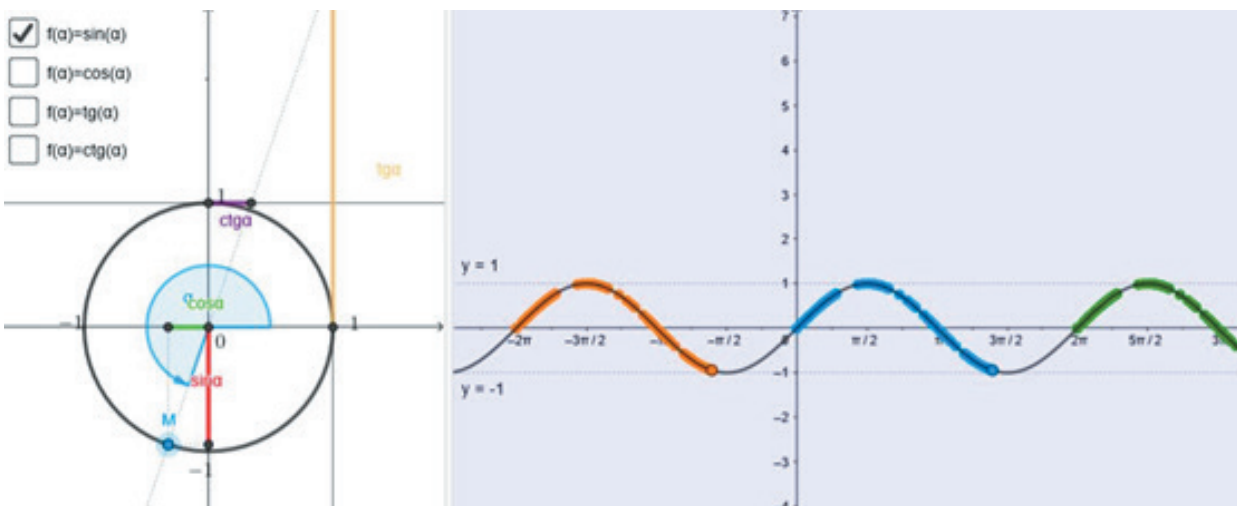

Figure 3 – Graphs of trigonometric functions.

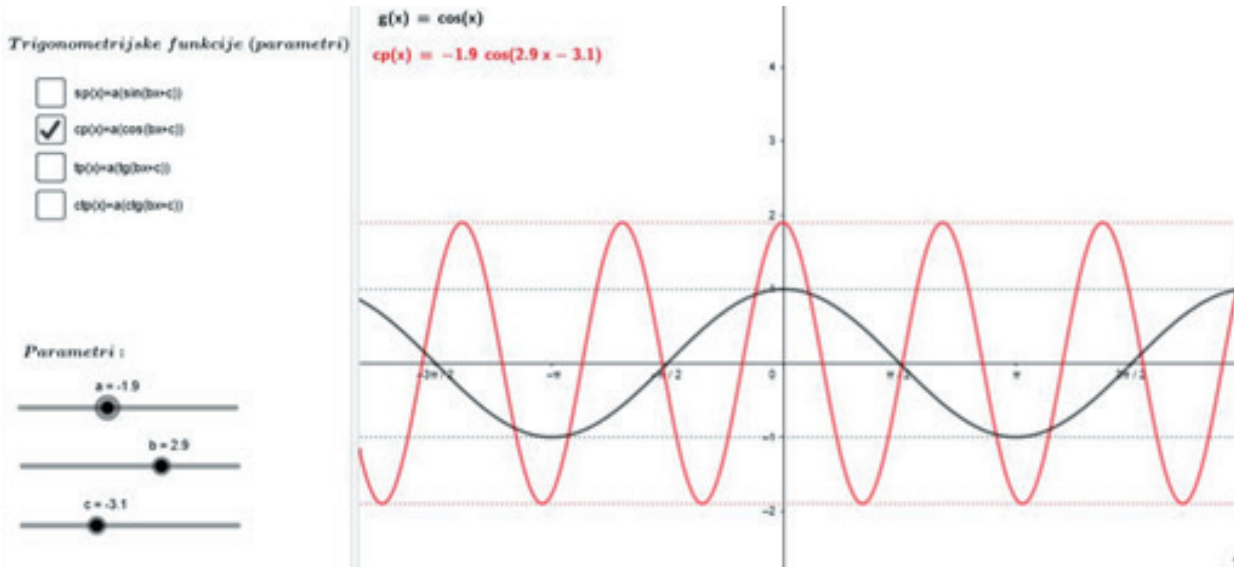

Figure 4 – Trigonometric functions with parameters.

3.5. TRIGONOMETRIC EQUATIONS

The GeoGebra material for trigonometric equations has checkboxes in the right window, one for each trigonometric function. The form of the trigonometric equation is f(x)=a (where f(x) can be sinx, cosx, tgx or ctgx). The unit circle in the left window is used for choosing the value a in the equation, e.g. for selecting the type of equation by moving the point M on the circle. Also, by including the unit circle in this material we intended to show a graphical representation of all solutions for each kind of equation, Figure 5.

By setting the desired equation form using a unit circle, the students can see its solution in the right window. The solutions are given in two ways, as values of angles on interval $[0,2\pi]$ and also in the general form below.

The most important goal of this material is for students to see how the solutions of trigonometric equations are connected and that they are periodically repeated.

4. RESULTS AND DISCUSSION

First, it is important to emphasize that the presented materials were available to all students and that they were already familiar with GeoGebra because they have used it before for learning mathematics as a supporting software tool.

During the online teaching process using written materials together with the links to the GeoGebra materials, we monitored students' reactions and their achievements in learning. Especially, the homework of each student was carefully examined, to detect eventual weak spots which needed further explanations.

The students' reactions to the GeoGebra materials were positive, they commented that these materials helped them to understand the concepts of trigonometry. They emphasized that the dynamical nature of GeoGebra, i.e. the possibility to manipulate the materials and create different angles, choose parameters' values and compare trigonometric functions was what they needed to be able to better understand trigonometry. The students also suggested to the teacher that they wish to use GeoGebra materials in the future, for learning other courses of mathematics.

The students from generation 2020 had the trigonometry course realized completely online, using only written and GeoGebra materials. We wanted to compare the achievements of students from this generation to the achievements of students from previous generations who traditionally learned trigonometry, in the classroom in the school.

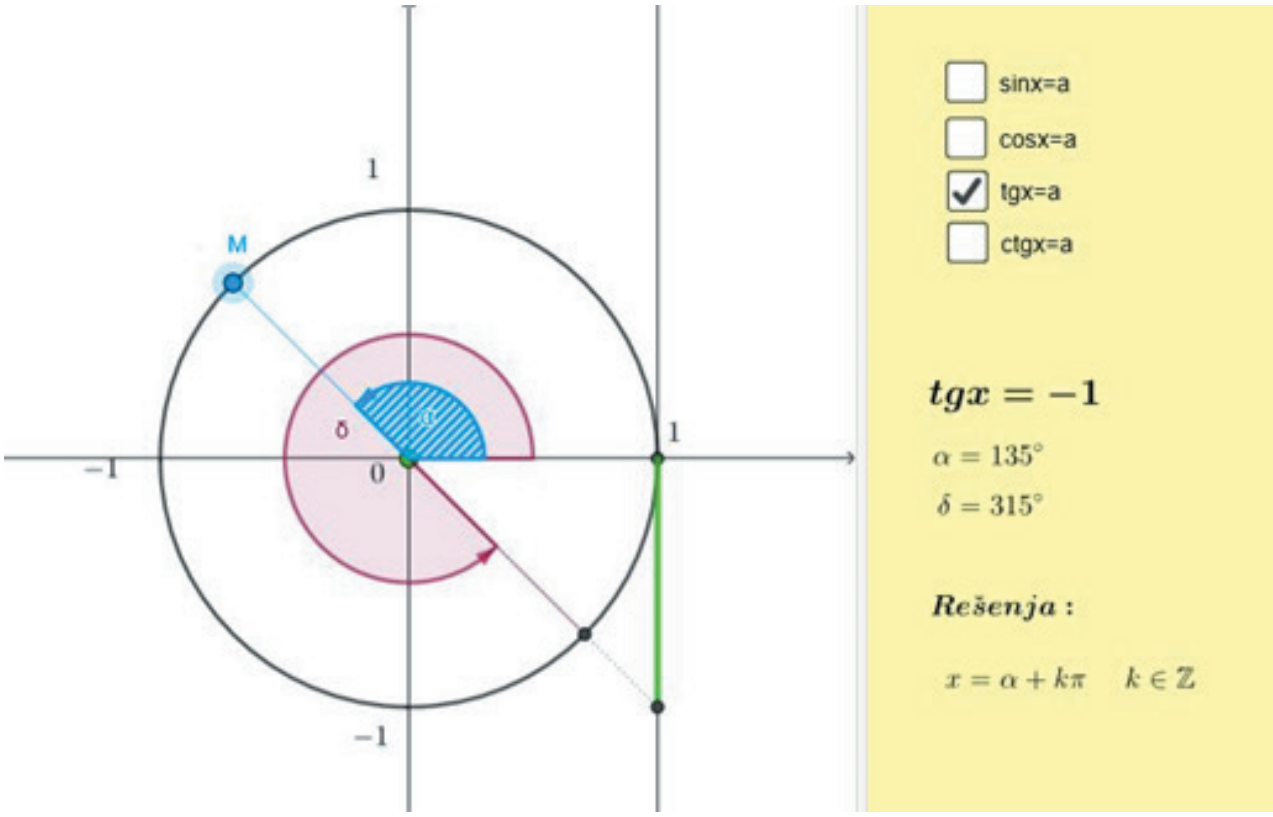

Figure 5 – Trigonometric equations.

We had the data on the results of the trigonometry test for generations of students from 2016 to 2020. All generations of students had similar tasks on the trigonometry test and all generations had the same mathematics teacher. The only difference was that the teaching process with generation 2020 was realized online and with the other generations in the classroom. We compared the percentage of the students' grades (from 5 to 1) on the trigonometry test, Figure 6.

Observing the grades of the 2020 generation, it can be noticed that (except for the generations 2018 and 2017 where exists a large disproportion of the highest grades among all other generations) the achievements of this generation is not particularly different compared to 2019 and 2016, or the average achievements of generations 2016 - 2019 in total (marked on the graph with 2016 – 2019 AVG), Figure 6. It implies that online teaching and using GeoGebra materials can contribute to teaching and learning trigonometry.

We also compared the average grade of all generations on the test. The average grade for generation 2020 was 3.40, while the average grade for generations 2016 – 2019 in total, was 3.58 which implies that students' achievements during the online teaching process did not fall behind the achievements of students who were taught in the classroom.

Although we have returned to the classroom after 2020, due to the positive experiences of both, students and teachers, we still use the GeoGebra materials presented in this paper as supporting materials for teaching trigonometry.

5. CONCLUSION

Teaching trigonometry courses in high school can be very challenging even in normal conditions, not to mention the online teaching process. However, in both cases, it is of great importance to find the best possible approach to present and explain trigonometry to the students.

Modern software tools allow teachers and students to explore various aspects of learning mathematics. GeoGebra software has proven to be particularly useful for teaching and learning mathematics because of its dynamic nature and the possibility for multiple representations of concepts.

We have presented and described the GeoGebra materials that we have created and used for teaching trigonometry courses in high school.

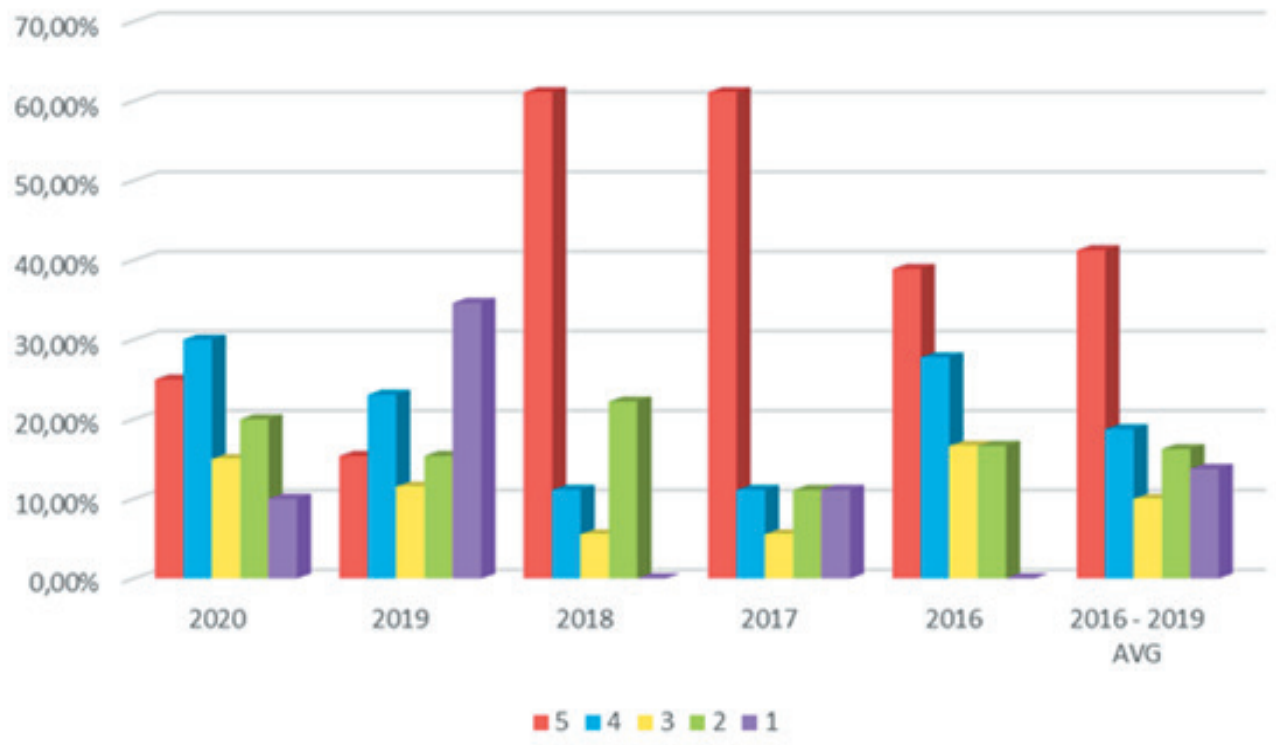

Figure 6 – The percentage of the grades on trigonometry tests for generations 2016 – 2020.

The teachers and the students had positive experiences using GeoGebra materials. The students' impressions about these materials were that they helped them to better understand trigonometric concepts, and most important of all, that they get the opportunity to experiment on their own using these dynamical materials.

The research results indicated that the online teaching process supported with GeoGebra materials managed to keep the level of students' achievements close to the level of the classroom teaching process.

Taking into account all the above-mentioned results and experiences of students and teachers, we can conclude that GeoGebra can be used to support both, the online and classroom teaching process and therefore, we continue to develop and improve GeoGebra materials for teaching mathematics at all levels of education.

6. REFERENCES

[1] N. Mosese and U. I. Ogbonnaya, "GeoGebra and students' learning achievement in trigonometric functions graphs representations and Interpretations," *Cypriot Journal of Educational Sciences*, vol. 16, no. 2, pp. 827-846, 2021.

[2] "GeoGebra," [Online]. Available: geogebra.org.

[3] T. Bekene Bedada and F. Machaba, "The effect of GeoGebra on STEM students learning trigonometric functions," *Cogent Education,* vol. 9, no. 1, 2022.

[4] C. Verhoef, F. Coenders, J. M. Pieters, D. van Smaalen and D. O. Tall, "Professional development through lesson study: teaching the derivative using GeoGebra," *Professional Development in Education*, vol. 41, no. 1, pp. 109-126, 2014.

[5] V. Kostić, V. Stankov Jovanović, T. Sekulić and D. Takači, "Visualization of problem solving related to the quantitative composition of solutions in the dynamic GeoGebra environment," *Chemistry Education Research and Practice*, vol. 17, no. 1, pp. 120-138, 2016.

[6] T. Sekulic, D. Takaci, M. Strboja and V. Kostic, "Influence of Mathematical Modeling in GeoGebra Environment on Learning Derivative," The International Journal for Technology in *Mathematics Education,* vol. 27, no. 2, pp. 61-82, 2020.

[7] K. Ibrahim and Y. Ilyas, "Teaching a concept with GeoGebra: Periodicity of trigonometric functions," *Educational Research and Reviews,* vol. 11, no. 8, pp. 573-581, 2016.

[8] K. Hernawati and H. D. Surjono, "Improving trigonometry learning motivation using Geogebra-Assisted Guided Discovery Student Worksheet (GDSW)," *Journal of Physics: Conference Series,* vol. 1320, p. 012082, 2019.