CLASSIFICATION, GEOMETRICAL AND KINEMATIC ANALYSIS OF FOUR-BAR LINKAGES

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Abstract:

This paper is dedicated to the classification of four-bar linkages, that is based on their geometric and kinematic characteristics. Depending on the type of links, which could be a crank, a rocker, a 0-rocker and a π-rocker, all four-bar linkages are divided into 27 groups. The proposed classification is a good basis for computer simulations that include motion analysis and motion visualization, which in turn could help to enhance students' understanding of this unit properly. Primarily, this paper has educational importance in the field of 3D modeling and synthesis of mechanisms.

Keywords:

mechanisms, four-bar linkages, education, computer simulations, motion analysis.

1. INTRODUCTION

Most machines, even very complicated ones, are built up from only a few types of commonly used mechanisms, such as linkages, gears, cam mechanisms, etc. The functions and profiles of machines may be quite different, but mechanisms used in them are often the same [1, 3].

Examples of four-bar linkages can be found almost everywhere around us and have wide range of application such as pumpjack, bycicle pedaling, suspensions with Watt's linkages, door closer, sewing machine, windscreen wiper, locking pliers. They could be found in human body (knee joint) and animal world (parrotfish jaw, Moray eel jaw) as well. These mechanisms consist of four bars or links and have one degree of freedom (DOF). One of the rotating members (links/bars) is called a driver or input link (*a*), and the other one a follower or output link (*b*). The floating link that connects input and output link is called a coupler link (*f*) and the fixed link is called a frame (*g*) [2]. Such a mechanism is considered to be one of the fundamental ones and it is shown in Figure 5. There are spherical four-bar linkages as well. The axes of four hinged joints are angled so they intersect in one point and links move on concentric spheres.

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2. CLASSIFICATION AND SIMULATION

Classification of four-bar linkages can be based on the type of movement performed by input link *a* and output link *b*. Each of them (both links *a* and *b*) rotate around hinged supports. However, this rotation can be either complete or partial. If a link performs a complete rotation (angle of 360°), then it is called *a crank*. If it performs only a partial rotation (total angles are less than 360°), this member is called *a rocker*. Generally speaking, each of these links *a* and *b* can be:

◆ a crank:	Figure 1; rotates completely (total angle is 3600); there is neither minimum (α_{min}), nor maximum (α_{max}) angle between link and *x* axis.
◆ a rocker:	Figure 2; rotates partially (total angle is less than 3600); there are both minimum (α_{min}) and maximum (α_{max}) boundary angle between link and *x* axis.
◆ a 0-rocker	Figure 3; rotates partially (total angle is less than 3600); there is not minimum (α_{min}), but there is maximum (α_{max}) boundary angle between member and *x* axis; link oscilates about 00, with amplitude ± α_{max}.
◆ a π-rocker	Figure 4; rotates partially (total angle is less than 3600); there is minimum (α_{min}), but there is not maximum (α_{max}) boundary angle between link and *x* axis; link oscilates about 1800, with amplitude (π ± α_{max}).

Table 1

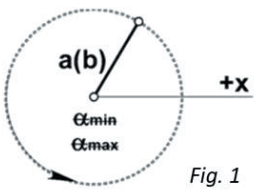


Fig. 1

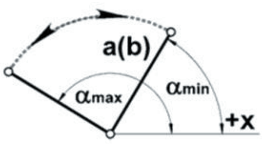


Fig. 2

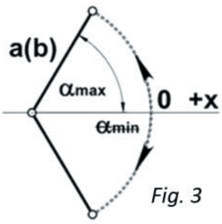


Fig. 3

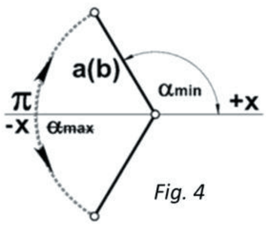


Fig. 4

Since the classification is based on existence of minimum and maximum (boundary) angles that are formed between input link *a*, output link *b* and *x* axis, it is crucial for these angles to be defined. They should be given as functions of links' length (*a, b, f, g*) as it is shown in Figures 5, 6, 7, 8, 9, 10, 11. According to the algebraic expressions, the existence of these angles will be discussed.

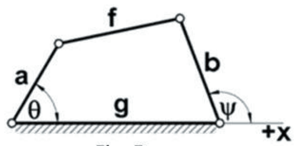


Fig. 5

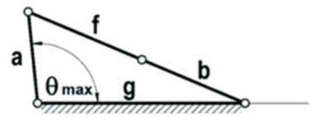
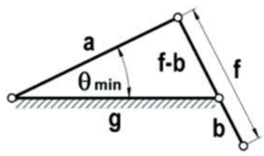


Fig. 6



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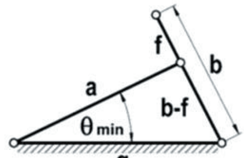

Fig. 8

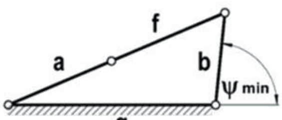

Fig. 9

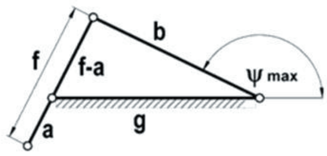

Fig. 10

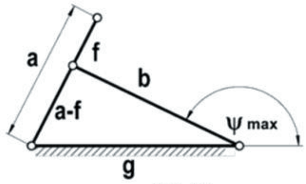

Fig. 11

Minimum and maximum angle that are formed between input link *a* and *x* axis are marked as θ_{min} and θ_{max} respectively. Configuration of four-bar linkages with maximum value of angle θ_{max} is shown in Figure 6, while in Figures 7 and 8 are shown configurations of this mechanism with minimum value of angle θ_{min} [4]. According to the Law of cosines, following relations are:

$$(f - b)^2 = (b - f)^2 = a^2 + g^2 - 2ag \cdot cos\theta_{min} ,$$

$$cos\theta_{min} = (a^2 + g^2 - (f - b)^2) / 2ag \quad \text{or}$$

$$cos\theta_{min} = (a^2 + g^2 - (b - f)^2) / 2ag ,$$

$$(f + b)^2 = a^2 + g^2 - 2ag \cdot cos\theta_{max},$$

$$cos\theta_{max} = (a^2 + g^2 - (f + b)^2) / 2ag \ .$$

Based on analysis of angle existence, input link *a* could be: a crank, a rocker, a 0-rocker and a π-rocker.

1. Input link *a* is a crank

$\cos\theta_{min} \geq 1$ and $\cos\theta_{max} \leq -1$;
(there is neither θ_{min} nor θ_{max})

In order to be more accurate, four expressions T_1, T_2, T_3 and T_4 should be defined:

$$T_1 = g+f-(a+b) \quad (1)$$

$$T_2 = b+g-(a+f) \quad (2)$$

$$T_3 = b+f-(a+g) \quad (3)$$

$$T_4 = a+g+f+b > 0. \quad (4)$$

The last one T_4 should always be positive ($T_4 > 0$). Expressions (1), (2), (3) and (4) are the same in all four cases of input link. Therefore, the input link *a* can be a crank only if two following relations are fulfilled:

$(-T_1) \cdot (-T_2) \geq 0$ or $T_1 \cdot T_2 \geq 0$;
$(-T_3) \cdot T_4 \geq 0$ or $T_3 \geq 0$.

2. Input link *a* is a rocker

$\cos\theta_{min} < 1$ and $\cos\theta_{max} > -1$;
(there are both of angles θ_{min} and θ_{max})

Hence, the input link *a* can be a rocker only if following relations are fulfilled:

$T_1 \cdot T_2 < 0$ and $T_3 < 0$.

3. Input link *a* is a 0-rocker

$\cos\theta_{min} \geq 1$ and $\cos\theta_{max} > -1$;
(there is not θ_{min}, but θ_{max} exists)

The input link *a* can be a 0-rocker only if following relations are fulfilled:

$T_1 \cdot T_2 \geq 0$ and $T_3 < 0$.

4. Input link *a* is a π-rocker

$\cos\theta_{min} < 1$ and $\cos\theta_{max} \leq -1$;
(there is θ_{min}, but there is not θ_{max})

The input link *a* can be a π- rocker only if following relations are fulfilled:

$T_1 \cdot T_2 < 0$ and $T_3 \geq 0$.

Minimum and maximum angle that are formed between output link *b* and *x* axis are marked as ψ_{min} and ψ_{max} respectively. Configuration of four-bar linkages with minimum value of angle ψ_{min} is shown in Figure 9, while in Figures 10 and 11 are shown configurations of this mechanism with maximum value of angle ψ_{max} [4]. According to the Law of cosines, following relations are:

$$(f+a)^2 = b^2+g^2-2bg\cdot\cos(\pi-\psi_{min}) = b^2+g^2+2bg\cdot\cos\psi_{min},$$

$$\cos\psi_{min} = ((f+a)^2-(b^2+g^2))\,/\,2bg,$$

$$(f-a)^2=(a-f)^2=b^2+g^2-2bg\cdot\cos(\pi-\psi_{max})= b^2+g^2+2bg\cdot\cos\psi_{max},$$

$$\cos\psi_{max} = ((f-a)^2-(b^2+g^2))\,/\,2bg \quad \text{or}$$

$$\cos\psi_{max} = ((a-f)^2-(b^2+g^2))\,/\,2bg\ .$$

As it was done in the case of input link *a*, classification of output link *b* can be made exactly the same. Relations (1), (2), (3) and (4) are valid in the case of driver (output link) as well.

1. Output link *b* is a crank

$\cos\psi_{min} \geq 1$ and $\cos\psi_{max} \leq -1$;
(there is neither ψ_{min} nor ψ_{max})

$(-T_2) \cdot T_4 \geq 0$ and $T_1 \cdot T_3 \leq 0$.

Therefore, the output link *b* can be a crank only if two following relations are fulfilled:

$T_2 \leq 0$ and $T_1 \cdot T_3 \leq 0$.

2. Output link *b* is a rocker

$\cos\psi_{min} < 1$ and $\cos\psi_{max} > -1$;
(there are both angles ψ_{min} and ψ_{max})

$(-T_2) \cdot T_4 < 0$ and $T_1 \cdot T_3 > 0$.

The output link b can be a rocker only if following relations are fulfilled:

$T_2 > 0$ and $T_1 \cdot T_3 > 0$.

3. Output link b is a 0-rocker

$\cos\psi_{min} \geq 1$ and $\cos\psi_{max} > -1$;
(there is not ψ_{min}, but the angle ψ_{max} exists)

$(-T_2) \cdot T_4 \geq 0$ and $T_1 \cdot T_3 > 0$.

Hence, the output link b can be a 0-rocker only if following relations are fulfilled:

$T_2 \leq 0$ and $T_1 \cdot T_3 > 0$.

4. Output link b is a π-rocker

$\cos\psi_{min} < 1$ and $\cos\psi_{max} \leq -1$;
(there is ψ_{min}, but the angle ψ_{max} does not exist)

$(-T_2) \cdot T_4 < 0$ and $T_1 \cdot T_3 \leq 0$.

The output link b can be a π-rocker only if following relations are fulfilled:

$T_2 > 0$ and $T_1 \cdot T_3 \leq 0$.

Since the indicators (T_1, T_2, T_3, $T_1 \cdot T_2$ and $T_1 \cdot T_3$) for classification of input link a and output link b of four-bar linkage are derived (Table 2), these mechanisms can be divided in 27 groups and it is shown in Table 3.

Input link *a*		**Output link *b***	
crank:	$T_1 \cdot T_2 \geq 0$; $T_3 \geq 0$	crank:	$T_2 \leq 0$; $T_1 \cdot T_3 \leq 0$
rocker:	$T_1 \cdot T_2 < 0$; $T_3 < 0$	rocker:	$T_2 > 0$; $T_1 \cdot T_3 > 0$
0–rocker:	$T_1 \cdot T_2 \geq 0$; $T_3 < 0$	0–rocker:	$T_2 \leq 0$; $T_1 \cdot T_3 > 0$
π–rocker:	$T_1 \cdot T_2 < 0$; $T_3 \geq 0$	π–rocker:	$T_2 > 0$; $T_1 \cdot T_3 \leq 0$

Table 2

No.	T_1	T_2	T_3	T_1T_2	T_1T_3	a	b
1	+	+	+	+	+	crank	rocker
2	0	+	+	0	0	crank	π–rocker
3	-	+	+	-	-	π–rocker	π–rocker
4	+	0	+	0	+	crank	0–rocker
5	0	0	+	0	0	crank	crank
6	-	0	+	0	-	crank	crank
7	+	-	+	-	+	π–rocker	0–rocker
8	0	-	+	0	0	crank	crank
9	-	-	+	+	-	crank	crank
10	+	+	0	+	0	crank	π–rocker
11	0	+	0	0	0	crank	π–rocker
12	-	+	0	-	0	π–rocker	π–rocker
13	+	0	0	0	0	crank	crank
14	0	0	0	0	0	crank	crank
15	-	0	0	0	0	crank	crank
16	+	-	0	-	0	π–rocker	crank
17	0	-	0	0	0	crank	crank
18	-	-	0	+	0	crank	crank
19	+	+	-	+	-	0–rocker	π–rocker
20	0	+	-	0	0	0–rocker	π–rocker
21	-	+	-	-	+	rocker	rocker
22	+	0	-	0	-	0–rocker	crank
23	0	0	-	0	0	0–rocker	crank
24	-	0	-	0	+	0–rocker	0–rocker
25	+	-	-	-	-	rocker	crank
26	0	-	-	0	0	0–rocker	crank
27	-	-	-	+	+	0–rocker	0–rocker

Table 3

Example 1: *crank-rocker four-bar linkage*; Since all three indicators T_1, T_2 and T_3 are positive, then the following products $T_1 \cdot T_2$ and $T_1 \cdot T_3$ are positive as well. In this case input link a can be only a crank because both conditions are fulfilled ($T_1 \cdot T_2 \geq 0$ and $T_3 \geq 0$) and output link b can be only a rocker ($T_2 > 0$ and $T_1 \cdot T_3 > 0$).

Example 2: *crank-π-rocker four-bar linkage*; In this case first indicator is 0 and other two are positive ($T_1 = 0$, $T_2 > 0$ and $T_3 > 0$). Products $T_1 \cdot T_2$ and $T_1 \cdot T_3$ are 0 and input link can be only a crank ($T_1 \cdot T_2 \geq 0$ and $T_3 \geq 0$) and output link can be only a π-rocker ($T_2 > 0$ and $T_1 \cdot T_3 \leq 0$).

Example 14: double crank four-bar linkage; All three indicators are 0 ($T_1 = 0$, $T_2 = 0$ and $T_3 = 0$) and both input and output link can be only cranks ($T_1 \cdot T_2 \geq 0$ and $T_3 \geq 0$; $T_2 \leq 0$ and $T_1 \cdot T_3 \leq 0$).

Example 19: 0-rocker-π-rocker four-bar linkage; First two indicators are positive and the third one is negative ($T_1 > 0$, $T_2 > 0$ and $T_3 < 0$), and two following products are $T_1 \cdot T_2 > 0$ and $T_1 \cdot T_3 < 0$. In this case input link can be only a 0-rocker ($T_1 \cdot T_2 \geq 0$ and $T_3 < 0$) and output link a π-rocker ($T_2 > 0$ and $T_1 \cdot T_3 \leq 0$) [5].

In the end, there are two conditions more that should be accomplished for synthesis of four-bar linkages. First one is existance of the mechanism as geometric figure, and the second one is Grashof's condition. It defines the existance of a crank as a link that performs complete rotation in four-bar linkage. The first condition is valid if following expression is fulfilled:

$$E = l - (s + p + q) \geq 0 \ , \tag{5}$$

where l represents length of longest link, s length of shortest link, p and q lengths of two remaining links.

If relation 5 is fulfilled, then this four-bar linkage can be constructed.

Grashof's condition is:

$$G = l + s - (p + q) \leq 0 \ . \tag{6}$$

If previously mentioned expression 6 is fulfilled, then link s (the shortest one) in four-bar linkage (a or b or f) can rotate completely for total angle of 360° and such a mechanism is named Grashof's linkage. If input link a and output link b are rockers (0-rocker and π-rocker) and Grashof's condition is fulfilled ($G \leq 0$), then the shortest link s is a coupler link f and as a crank preforms complete rotation for total angle of 360°.

Example 21 (Table 3); such mechanisms are named Grashof's double rocker linkages. A part of this group of mechanisms is Chebyshev linkage (for g: a: b: f = 4: 5: 5: 2) as well.

If Grashof's condition is not fulfilled ($G > 0$), all links are rockers. There is one boundary case when $G = 0$. These mechanisms are named Grashof's neutral linkages. There is one link that rotates for total angle of 360°, but mechanism has a singular configuration, in which the movement of output link is not precisely defined.

Grashof's condition can be replaced by product of three indicators T_1, T_2 and T_3 ($T_1 \cdot T_2 \cdot T_3$). If the product is positive ($T_1 \cdot T_2 \cdot T_3 > 0$), then G is negative ($G < 0$) and this four-bar linkage is Grashof's linkage. This condition is suitable when lenghts of links l and s are not known [6].

3. CONCLUSIONS

Theoretical lecture mode adopts the basic theories of mechanisms and provides an interactive learning style for users via an interactive tool such as computer simulations [3]. One of the programmes for modeling and simulations used by students of Faculty of Mechanical Engineering in Belgrade is *SolidWorks*. In order to make a proper model of a mechanism and to simulate its motion, they need theoretical knowledge and this paper deals with ease approach to it. The model of four-bar linkage made according to previously mentioned kinematic and geometrical analysis is shown in Figure 12. A detailed analysis of four-bar linkages has proved to be very useful for students and understanding of this unit has quite enhanced.

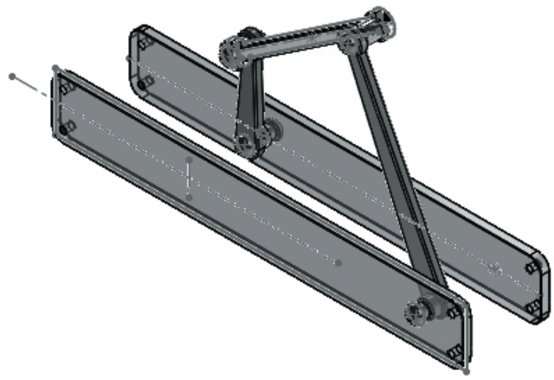

Fig. 12

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