



TIMELINE OF MEDIAN FILTER

VREMENSKI OKVIR MEDIJAN FILTERA

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Abstract:

This research paper gives a chronological overview of the median filter development, as one of the filters recommended for digital image processing. It takes into account the patents based on median filter, various published research and development ideas presented at conferences, as well as the paperwork, including parts of books and journals, whose main topic is this filter, starting from the mathematical model of the filter to its practical applications in the field of digital image processing. In order to better understand this issue, some works are presented as sublimation of several different solutions in this area.

Key words:

median filter, digital image processing, digital signal processing.

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1. INTRODUCTION

Conceptual design on the elimination of noise from digital images based on averaging matrix defined through submatrix within an image have the widest application in the field of digital image filtering based on the spatial analysis. The original idea came from the analysis of the observed statistical areas of some environments, which are afterwards applied in reconstruction of the analogue and digital signals. The entire field of digital signal processing has accepted this idea and adapted to its needs.

Throughout its development, the median filter is used as a reference in 2625 papers published at various meetings or conferences, in 682 different journals, as well as four books. The fact that this year is the 40th anniversary of median filter gives the possibility to review research done on the development from the mathematical model of the filter to practical applications in field of Digital Image Processing (DIP).

2. THE BEGINNINGS

The story about filter that plays the crucial role in the area of Digital Image Processing (DIP) started in 1972, and is closely related to the idea firstly promoted by a group of scientists led by the professor John Turkey in the book "Robust Estimates of Location: Survey and Advances" (Andrews *et al.*, 1972). The basic idea proposed in the book was not intended for the area digital signal processing, but the description of algorithm was described in the chapter Linear combination of statistical order and M-estimation.

Apstrakt:

Ovaj rad daje hronološki prikaz razvoja medijan filtera kao jednog od preporučenih filtera za digitalnu obradu slike. U radu su prikazani patentni zasnovani na medijan filteru, razna publikovana istraživanja i ideje prezentovane na konferencijama, kao i delovi knjiga i časopisa koji se bave ovim tematikom, počevši od originalnog matematičkog modela medijan filtera do njegove praktične primene u oblasti digitalne obrade slike. U cilju što boljeg razumevanja ove problematike i za potrebe ovog rada, pojedini radovi i istraživanja predstavljeni su kao sublimacija nekoliko različitih rešenja u ovoj oblasti.

Ključne reči:

medijan filter, digitalna obrada slike, digitalna obrada signala.

Turkey' gave the so-called theory Median-Median line (1974), which states that some certain odd matrix or series of numbers can have a potential of average value (median value) in the central element if sorted by value- from the lowest to the highest. That further implies that if there is a series of number such as: 2, 4, 1 and if they are sorted as 1, 2, 4 – median value in these series of number is 2.

Almost at the same time, in two separate studies, the proposition related to the Turkey' theorem implementation was made. The first one from 1976 was related to noise reduction in speech signals during the transmission (Jayan *et al.*, 1976), and the second one from 1975 gave a proposition related to median implementation as an algorithm of digital image processing (Pratt, 1975). The point was to divide image into sub-matrix 3x3, to sort those sub-matrix and add value of central pixel to the complete sub-matrix. The main problem of this algorithm was hardware's complexity even though images were processed in low resolution Grayscale mode. Even though there was a lot of imperfections and without broad implementation, this filter version will be the bedrock for many subsequent comparisons and updates.

In the following period, in 1976, the conceptual solution was found. It can be said that this solution to some extent reduced errors of the basic version of the filter edges (Frieden, 1976). In this paper, median filter was presented as a non-linear function over the monochromatic image. However, this version was even more complex for realization than the previous one. That was the reason why optimization was a priority. After three years, in the middle of 1979, and with the use of fast median algorithm



and definition of median filtering masks, the complexity of the filter is significantly reduced.

A fast median filter was characterized by the usage of sub-matrix histogram, by which median value could be defined, after which the vertical scrolling will be done and over and over new histogram value shall be determined (Huang & Tang, 1979). On the other hand, definition of masks for median filtration defined non-linear filter through the median absolute deviation (MED). In this case, spatial variable will be moved in image and that way each MED for sub-matrix will be defined (Gray *et al.*, 1979). The paper was practically a result of a combination of paper works (Huang & Tang, 1979; Gray *et al.*, 1979) was based on the fast method of compression by median filter in real time (Ataman *et al.*, 1980), and compression bit by bit. This paper reduced not only hardware complexity but also software's complexity.

In 1981, Mr. Patrenahalli M. Narendra in the IEEE journal was talking about median filter as a "separate 2D filter", or as he defined it "one dimension filter by x and y direction" (Narendra, 1981). In practice, that meant that the image now can be seen through the spatial domain, and image processing can be done in x and y direction, which gave different point of view on the area that just started to develop.

Three years after this original idea, one experimental realization took place at the University of Rochester, as a result of cooperation between the department of statistics and computer science. This realization gave a solution for additional mitigation of the level of noise in digital image. Basis of this paper work was consisted of usage of one dimensional series several times in order to obtain two dimensional series that is operational in sub-matrix $4\log_2(n+1)-2$ or square sub-matrix $n \times n$ (Basu & Brown, 1984).

From the current perspective, it can be said that the lack of this algorithm is in square resolution of sub-matrix, which means that in the analyses process an error in images without square resolution (4:3, 16:9 and 16:10) must be taken into consideration. However, what is different in this algorithm is the introduction of different degrees of sub-matrix, i.e. 3×3 , 5×5 , 7×7 , 11×11 , 13×13 , etc. Accordingly, the algorithm became more accessible for a wide range of users because the optimization made it less complex for processors. Complexity was linearly reduced by the level of sub-matrix implementation. In the same year, in paper work (Brownrigg, 1984). conceptual solution for image transmission was given by name the weighted median filter in which the value of sub-matrix had predefined value, their transmitting side matrix would be divided and on the receiver side it would be multiplied.

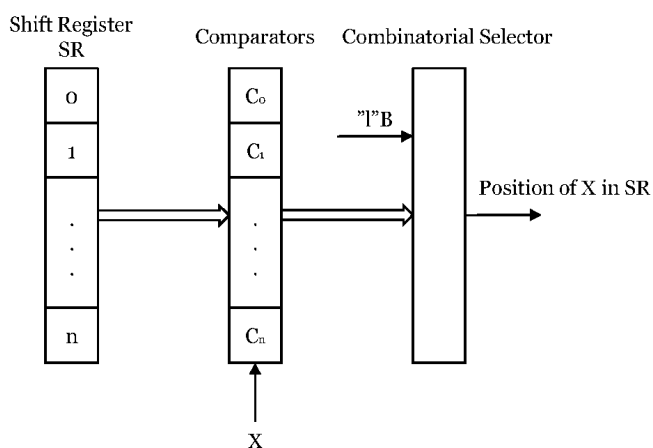


Figure 1. Hardware implementation of median filter from 1984
Hardware implementation from paper published in 1984

is shown in Fig. 1. Realization is performed via shift register (Shift Register - SR), which can receive n pixels. This register can be represented as a vector, while the shifting operation is realized by moving from left to right. The second part is implemented via circular BX register (comparator) that searches all n elements sorted in SR and outputs a value $BX(X)$ which is the central value of the sub-matrix as defined by the median filter (Pratt, 1975).

A total of 16 years has passed from the basic idea until the first practical realization. At the beginning of 1988, William T. Freeman, from Polaroid Corporation, registered patent entitled "Median filter for reconstructing missing colour samples" (Patent US 4724395 A, 1988). Products called "electronic cameras for creating colour images conventionally" had a high level of noise in output signal, and thus median filter implementation was necessary. Specifically, sensors that were used included the entire range of wavelengths of visible spectrum, and since they did not react identically to the level of light at a particular wavelength, certain parts of the spectrum were reproduced less than others, in the output signal. The conceptual design presented in the patent involved separation of the visual range in three areas, each processed with a median filter, respectively, as shown in Fig. 2.

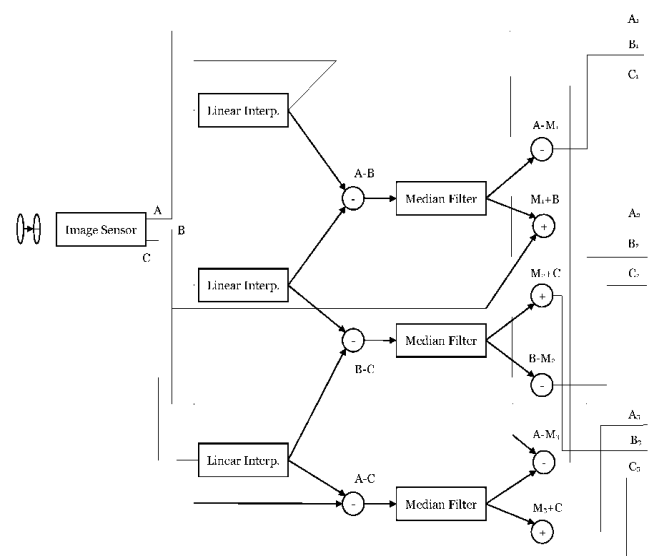


Figure 2. Implementation of median filter in first digital camera

Figure 2 presents image sensor that gives three characteristic signals at the output, for three specific wavelengths that correspond to the colours. The signal strength corresponds to the strength of the light, on which sensor reacted. In this case, the problem was that a sensor has not been sensitive completely to all of the three observed wavelengths. In order to solve this problem partially, linear interpolator is introduced, for each of the three separate signals. Thereafter, signal difference is performed corresponding to the brightness difference between the inputs signals, then each of the individual signals are subjected to median algorithm. The signals obtained at the output are presented and they match to colours most sensitive to human eye: red, green and blue (RGB).

Another patent was significant for the year 1988, and it was a prediction algorithm for median filter. All previous solutions were based on a square sub-matrix and the noise elimination using them. Thus, the idea not to use previously developed algorithms for filtering, and take the basis proposed by the Tukey was the reason for a new patent.

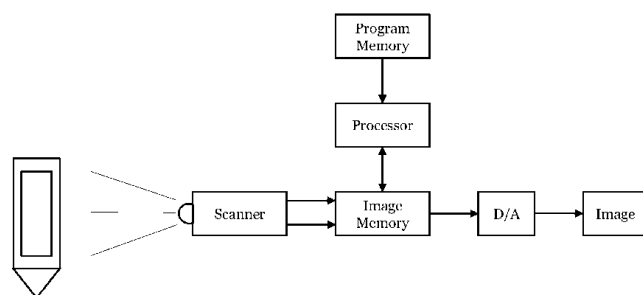


Figure 3. Block diagram of the patent
Source: Source: Patent US 4736439 A (1988).

Patent block diagram shown in Fig. 3 indicates that the signal obtained from the image sensor is stored in the image memory managed by the processor. Filter algorithm is located in program memory and processor uses it over the loaded images. Upon completion of the operation, the signal is submitted to the D/A converter whose output may actually be the input of a monitor. In this case, it is very interesting to have an insight into the spatial reconstruction of the algorithm, to differentiate it from the standard operations. One of the real situations is shown in Fig. 4.

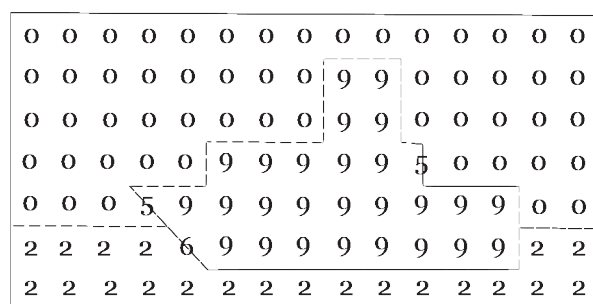


Figure 4. Realistic display of spatial analysis presented in the patent
Source: Patent US 4736439 A (1988).

As can be seen in Fig. 4, the area being observed (within the dashed lines) is not a square shape, but the pixels within a certain range of variation (in this case ± 4) consist of sub-matrix to which a median filter is applied.

At the end of the 80s, rapid development of commercial digital cameras could be observed, which hit the market ten years later. Accordingly, with this and similar patents, Polaroid Corporation has set itself up as a respectable company in the following years (The Washington Post, 2012 a; The Washington Post, 2012 b).

In 1991, NASA recognized the ability of this filter and started using it in geo-physics, *i.e.*, in wind recordings that are recorded on the surface of the ocean. Rate of recordings of satellites was divided into polygons with the width rate of 350 km divided in two columns of 175 km, while the columns were further divided into square polygons defined by vector direction. Real situation obtained after the recordings and divided into two columns with predefined polygons is presented in Fig. 5. On the basis of NSCAT system for data processing obtained on the Earth, surface vectors in polygons were defined. As can be seen in the image, the direction of vectors obtained polygons are chaotically distributed, which makes it more difficult to draw a unique conclusion.

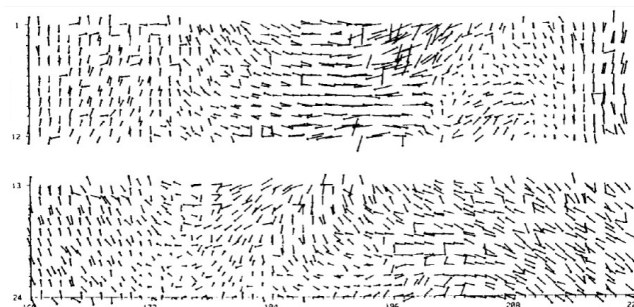


Figure 5. Realistically obtained satellite images (range - 350 km)

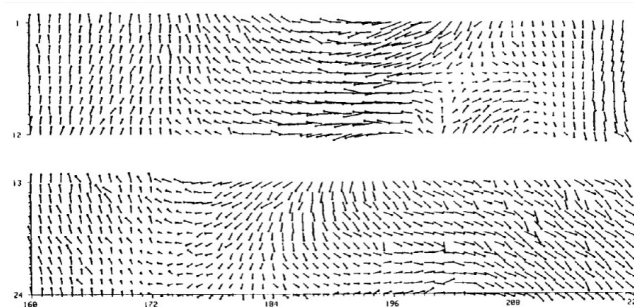


Figure 6. The obtained polygons after median filter applying

The implementation of median algorithm used on the obtained satellite recordings is presented in Fig. 6. The study done in 12 different world regions with different wind concentration and vector reconstruction inside the polygon by filter in comparison with ideally assumed image was from 96.7% to 98.69%. In order to get better quality optimization with polygon, sub-matrix 7x7 was used. This led to a conclusion that vector algorithm showed a high level of stability in the area that primary was not intended to.

3. THE NEW CONCEPT

Taking into account the above-stated, median filter, in the mid-nineties, clearly laid the foundations of the areas in which they had expected contribution. Therefore, papers published in those years aimed at certain specific improvements in which they could act. If we note that Polaroid patent was still in force, it can be concluded that the area of colour correction was limited when the survey was concerned. For example, paper (Florencio& Schafer, 1994), provides a statistical analysis of the results in the processing of the specific parts of the image (especially those with edges), while focusing only on the impulse type of noise and give specific algorithmic solutions. The emphasis of the given algorithm is on using identity filters besides median filter, but in situations with images containing large number of edges, resulting image provides up to 2 dB better quality than other reconstruction techniques. However, one of the conceptual designs was the use of a filter as a prediction of some edge detection algorithms or compressions as presented in (Neuvo, 1994). In addition, median filter is also applied in fuzzy algorithm adapted for the use in multiple levels, something similar to implementation in the pyramidal algorithm for edge detection (Wu & Li, 1997). Filter version adapted to a larger number of edge detection algorithms is called MultiLayer modified Median Filter (MLMF) and is optimized specifically for the elimination of Salt & Pepper noise (Yang & Toh, 1995).



Figure 7. a) Original image supplemented with noise, b) reconstructed images with RAMF algorithm, c) reconstructed images with SAMF algorithm

Source: <http://www.ijettcs.org/Volume2Issue1/IJETTCS-2013-01-11-014.pdf>

In those years, a number of papers dealt with the definition of different types of digital image noise, so it is possible to assume the direction of the filter development. Based on the two types of defined categories of digital image impulse noise, two adaptive median filters are defined, which are called ranked-order based adaptive median filter (RAMF) and impulse size based adaptive median filter (SAMF). RAMF separately treats positive and negative impulse noise, while SAMF focuses only on removing high density impulse noise (Hwang & Haddad, 1995). The results of this algorithm are given in Fig. 7.

Almost all digital cameras in the late nineties were based on CCD sensors. However, a major disadvantage of these sensors is generation of high level of noise when shooting in high detail. This was manifested in situations while using the optical zoom, the image area with a high level of detail had a far greater degree of noise than other parts of the image. In such cases, the use of non-linear filters during reconstruction is expected. In addition, it was evident that the problem of filtering should be done before creating the image, because the noise density had uneven distribution on the image. This problem is solved with the so-called frequency selected median filter (frequency selective weighted median - FSWM), in which zoom level is related to the degree of sensitivity of filter via correlation function of the weighted median algorithm (Choi, 1999).

The end of 1999 is marked by another patent designed for elimination of positive and negative impulse noise. The idea of patent is based on the separation of the observed part of signal in groups and their definition on a circle, as shown in Fig. 8.

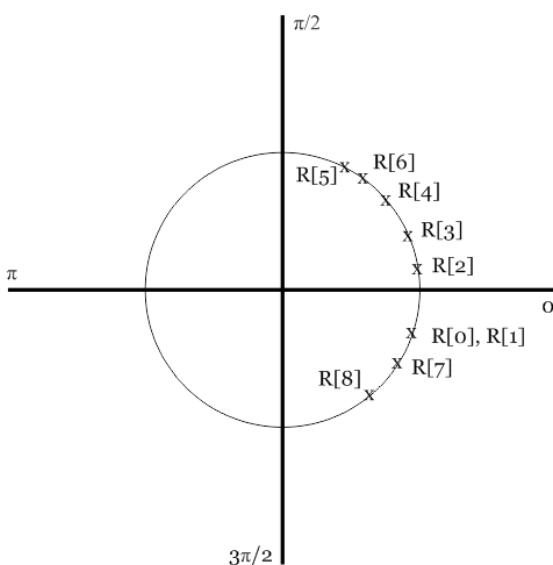


Figure 8. Defining values on a circle for observed signal

In this way, the value of impulse noise is defined on the basis of the maximum angle of inclination between the point and x-axis. Thus, the highest values were in the first quadrant for positive impulse noise, and in the fourth quadrant for negative impulse noise. In this case, filter processing is focused only on the points with the maximum angle of inclination (Patent US 5968111 A, 1999). Unlike CCD sensors, CMOS sensors are rarely used for image creation, because they are very susceptible to various types of noise. A small revolution in the field gives the patent about circular analogue median filter for digital image creation, and the use of CMOS technique in image creation increased in following years (Patent US 6121817 A, 2000).

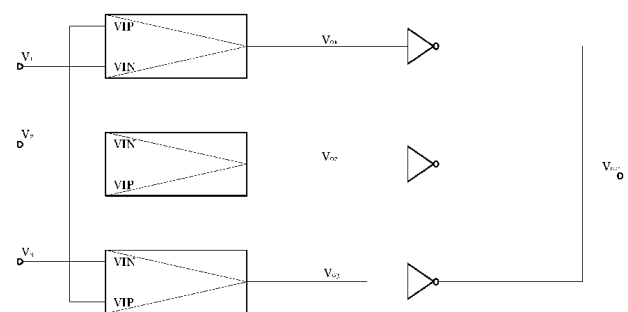


Figure 9. The implementation of circular analogue median filter

Source: <http://www.google.com.ar/patents/US6121817>

Conceptual realization, illustrated in Fig. 9, shows a way in which the signal received from the CMOS sensor and converted into three signals (RGB) is transmitted to transconductive amplifier, while eliminating extremes of impulse noise in all three signals. However, inverted signals are on the output of these amplifiers, so addition of three inverters is needed for signals to be returned.

4. MODERN CONCEPT

In 2001, Polaroid company withdrew from digital camera market, due to numerous problems, and thus the patent for colour correction ceased to be effective (The Washington Post, 2012b). However, in the following years, the application of median filter was limited mostly to works based on various analyses and definitions of different types of noise in the digital image, as well as the possibilities of filter optimization for specified noise. One of the solutions is proposed in the IEEE journal and is based on the analysis of digital image parts and image noise



prediction. Algorithm is used to decide what will undergo the Laplace filter and what median filter (Zhang & Karim, 2002).

Besides impulsive noise, the observed filter gives very good results in elimination of the so-called Salt-and-Pepper noise in digital image. Although it is often difficult to mathematically separate high level of detail and high level of noise in the same segment of the image, the paper from the year 2005 gives a very practical solution in this area, such as division of complete algorithm process into two phases: the first phase, in which it detects pixels - potential noise carriers, and the second phase in which it eliminates only the selected pixels (Chan *et al.*, 2015). In certain circumstances, such reconstruction gives remarkable results for 70% of image damage.

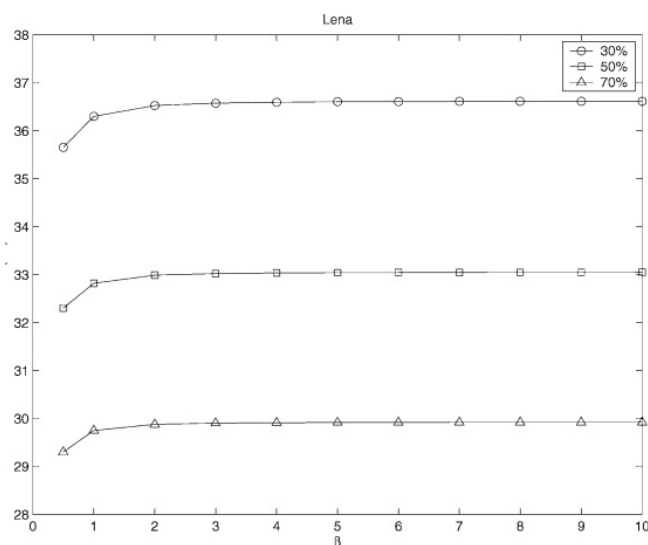


Figure 10. The values of PSNR parameter during the Salt-and-Pepper noise increase density

Fig. 10 gives an insight into the change of PSNR parameter for different degrees of noise density (30%, 50% and 70%). As can be seen, this algorithm gives extensive stability processing different levels of noise, around 90%, expressed in percentage.

Paper, directly correlated to (Chan *et al.*, 2015), was published three years later. Despite all the advantages, it used fuzzy algorithm in combination with median filter in order to eliminate Salt-and-Pepper noise from very complex images such as texture images. The main idea implied prediction (consisting of fuzzy algorithm) and the system of separate polygons for median filter (Toh, *et al.*, 2008). After a large number of papers in this area and other design solutions of digital image filtering, it is important to compare, by precisely determined parameters, nonlinear filters in the process of removing different types of noise. A qualitative analysis is given in (Church *et al.*, 2008), with the comparison of six most used versions of median filter in removing different types of noise.

Figure 11 shows the results of the comparative analysis of the six most commonly used versions of median filter for eleven different types of noise. Based on the results, the best filter is described in (Neuvo, 1994).

In 2010, Intel, as a leader in microprocessor technology, incorporates a median filter and optimizes it in the context of instructions specifically defined for work with multimedia (SSE instruction) (Intel, 2010). This process can be seen as a reward for all the results that this filter has achieved from 1975, as well as for great expectations from this filter in the future. Therefore,

the filter is optimized and easier to operate, especially as of 2010 when high-resolution images appeared and were made available to a large number of digital image users.

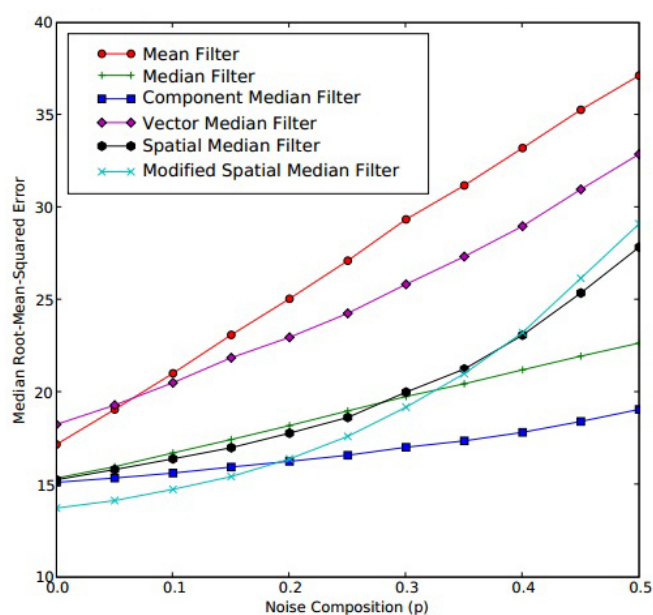


Figure 11. Comparative analysis of six versions of median filter across 11 noise models

This optimization by Intel enabled the combination of two median algorithms without major hardware requirements for Salt-and-Pepper noise elimination (Esakkirajan *et al.*, 2011). Similarly, this optimization enabled combination of median filter presented as linear and non-linear filter in removing different types of noise (Agrawal & Verma, 2013), or highly optimized noise reduction algorithms such as (Narayanan *et al.*, 2013; Vijaykumar, & Santhanamari, 2014). Today, the emphasis is on the high degree of optimization that can be found in publications from 2015, such as (Kalra & Singh, 2015). Image is seen as three completely separate signals, so by application of different versions (and combination of different versions) of median filter, depending on the properties of the observed signal, the desired reconstruction is obtained.

5. CONCLUSION

By observing the entire period of 40 years, it can be said that median filter is one of the basic pillars in the area of digital image processing, especially in the narrow area, where a wide range of usage of filters, related works, patents and published books indicates its importance. It is important to highlight that the expectations concerning the usage of this median algorithm in the following period are great, especially in the field of image reconstruction, which will be a great challenge in the upcoming years. The same process has already been done with blur algorithm.

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