Impact of Internet on Business Activities in Serbia and Worldwide
Uticaj Interneta na poslovanje u Srbiji i svetu

DOI: 10.15308/SINTEZA-2014-965-968

STATISTICAL CHARACTERISTICS OF SIGNAL IN THE PRESENCE OF TWO FAST NAKAGAMI-M AND ONE SLOW GAMMA FADING

Boško Radovanović[1], Petar Spalević[1], Stefan Panić[2], Mihajlo Č. Stefanović[3], Dejan Milić[3]

[1]Faculty of Technical Science Kosovska Mitrovica
[2]Faculty of Natural Science and Mathematics Kosovska Mitrovica
[3]Faculty of Electronics Engineering Nis

Abstract:
In this paper is described model of forming equivalent envelope of a signal at the entrance of the wireless digital telecommunication system when there are present simultaneously two fast Nakagami-m and one slow Gamma fading. For the described model probability density function (PDF) of random variable at the reception side has been determined. Dependence of the probability density function (PDF) of the parameters of fading (fading severity) and corresponding components power is analytically and graphically described. Influences of the fading parameters on statistical characteristics of the received signal and quality of the signal detection in the entire communication system has been graphically shown.

Key words:
fading figure,
Nakagami-m fading,
Gamma fading,
probability density function (PDF),
Outage probability.

INTRODUCTION

In wireless telecommunication systems the signal is exposed to so many different influences of the environment in which telecommunication channel is formed. Imperfection of electronic components, atmospheric conditions, electric discharge and other electromagnetic waves as well ground configuration influence the signal by degrading it. Every change in signal quality is treated as disturbance that is manifested on reception side. Disturbance might be noise, interference and fading.

Besides noise, fading has the biggest influence on radio signal. Fading occurs due to several reasons. In most of the case it is consequence of different conditions in radio channel. The ideal case related to signal transfer is when there is a straight line between transmitter and receiver *(Line Of Sight, LOS)*. However, that is not often the case, so signal on reception depends on signals arriving using different paths with different attenuation, phase shift and delays. By superposition of these radio signals at the reception, a signal is received with amplitude and phase changing in time. For such signal we say that it is influenced by fading [1-2].

Since the conditions in channel significantly change during symbol duration, this fading is called fast fading (multipath, short-time fading)[3-4]. On the other hand, in some cases receiver becomes interrupted, and signal power becomes attenuating or completely disappears at the reception. For such signal we say that it is influenced by shadowing effect (long-term fading). Since the conditions in channel happen occur more slowly, this fading is called also long-term fading [1]. Hence, signal at the entrance into receiver is influenced by fast and slow fading, which change its amplitude and/or power.

Due to importance of fading influence on radio signal, this document will consider performances of wireless digital telecommunication system in the presence of fading.

For the needs of analysing transmission characteristics of telecommunication systems, different distributions are used by which fading is modelled in different propagation environments.

For modelling of fast fading in most of the case Rayleigh, Rice, Nakagami-m and Weibull distribution is used, and for modelling of Slow fading Gamma distributions is used and it is significantly simpler from Lonormal distribution [1-2]. *Rayleigh* model is used for describing propagation conditions when there is no *LOS*. *Rice or Rician* model is used for description of propagation conditions when there is a dominant signal on *LOS*. Nakagami-m model represents general channel model with fading that can be reduced to other models for different Nakagami distribution parameters [5].

Chapter II explains statistical characteristics of signal at the entrance of the wireless digital telecommunication system when there are present simultaneously two fast and one slow fading. Fast fading will be modelled by Nakagami-m distribution, while slow fading will be modelled by Gamma distribution. In that way general channel model with fading is obtained. Display of numeric results obtained for different fading severity (m) and different ratio of signal power (Ω) at the entrance is given in chapter III. Analysis of obtained results and concluding considerations are given in chapter IV. At the end there is a preview of literature used.

MODEL OF CHANNELS

A case is considered when at the entrance into receiver of wireless telecommunication system there are two fast and one slow fading. Two fast and one slow fading influence on useful signal anvelope. In that case signal anvelope might be presented as a product of three random processes:

$$z = x\, y\, w \tag{1}$$

where the random process x describes fast fading and its envelope is distributed with Nakagami-m probability density function (PDF) [6]:

$$p_x(x) = \frac{2}{\Gamma(m_1)}\left(\frac{m_1}{\Omega_1}\right)^{m_1} x^{2m_1-1} e^{-\frac{m_1}{\Omega_1}x^2}, x \ge 0 \tag{2}$$

and random process y describes fast fading and its envelope is also distributed with Nakagami-m probability density function (PDF)

$$p_y(y) = \frac{2}{\Gamma(m_2)}\left(\frac{m_2}{\Omega_2}\right)^{m_2} y^{2m_2-1} e^{-\frac{m_2}{\Omega_2}y^2}, y \ge 0 \tag{3}$$

and random process w models shadowing effect and is distributed with Gamma PDF as [7]:

$$p_w(w) = \frac{1}{\Gamma(c)}\left(\frac{c}{\Omega_3}\right)^{c} w^{c-1} e^{-\frac{c}{\Omega_3}w}, w \ge 0 \tag{4}$$

By transformations of probability densities and by appropriate averaging we get *PDF* of equivalent envelope.

Since random variable x is

$$x = \frac{z}{y\, w} \tag{5}$$

conditional PDF of random variable z can be obtained according to:

$$p_z(z/yw) = \left|\frac{dx}{dz}\right| p_{x\dot{x}}\left(\frac{z}{y\, w}\right) \tag{6}$$

where $\left|\frac{dx}{dz}\right| = \frac{1}{y\, w}$.

By replacement we get

$$p_z(z/w) = \frac{1}{y\, w}\, p_x\left(\frac{z}{y\, w}\right) \tag{7}$$

PDF of equivalent envelope, could be now obtained by transforming:

$$p_z(z) = \int_0^\infty dy \int_0^\infty dw\, \frac{1}{y\, w}\, p_x\left(\frac{z}{y\, w}\right) p_y(y)\, p_w(w) =$$

$$= \int_0^\infty dy \int_0^\infty dw\, \frac{1}{y\, w}$$

$$\frac{2}{\Gamma(m_1)}\left(\frac{m_1}{\Omega_1}\right)^{m_1}\left(\frac{z}{y\, w}\right)^{2m_1-1} e^{-\frac{m_1}{\Omega_1}\frac{z^2}{y^2w^2}}$$

$$\frac{2}{\Gamma(m_2)}\left(\frac{m_2}{\Omega_2}\right)^{m_2} y^{2m_2-1}\, e^{-\frac{m_2}{\Omega_2}y^2}$$

$$\frac{1}{\Gamma(c)}\left(\frac{c}{\Omega_3}\right)^{c} w^{c-1}\, e^{-\frac{c}{\Omega_3}w} =$$

$$= \frac{2}{\Gamma(m_1)}\left(\frac{m_1}{\Omega_1}\right)^{m_1} z^{2m_1-1}\, \frac{2}{\Gamma(m_2)}\left(\frac{m_2}{\Omega_2}\right)^{m_2} \frac{1}{\Gamma(c)}\left(\frac{c}{\Omega_3}\right)^{c}$$

$$\int_0^\infty dy \int_0^\infty dw\, \frac{1}{y\, w}\left(\frac{1}{y\, w}\right)^{2m_1-1} y^{2m_2-1}\, e^{-\frac{m_2}{\Omega_2}y^2}$$

$$w^{c-1}\, e^{-\frac{c}{\Omega_3}w} \sum_{i=0}^{\infty}(-1)^i \left(\frac{m_1}{\Omega_1}\right)^i \frac{1}{i!}\, z^{2i}\, y^{-2i}\, w^{-2i} =$$

$$= \frac{4}{\Gamma(m_1)\Gamma(m_2)\Gamma(c)}\left(\frac{m_1}{\Omega_1}\right)^{m_1}\left(\frac{m_2}{\Omega_2}\right)^{m_2}\left(\frac{c}{\Omega_3}\right)^{c}$$

$$z^{2m_1-1}\sum_{i=0}^{\infty}(-1)^i\left(\frac{m_1}{\Omega_1}\right)^i \frac{1}{i!}\, z^{2i}$$

$$\int_0^\infty dy\;\, y^{-1-2m_1+1+2m_2-1-2i}\, e^{-\frac{m_2}{\Omega_y}y^2}\int_0^\infty dw\; w^{-1-2m_1+1+c-1-2i}\, e^{-\frac{c}{\Omega_3}w} =$$

$$= \frac{4}{\Gamma(m_1)\Gamma(m_2)\Gamma(c)}\left(\frac{m_1}{\Omega_1}\right)^{m_1}\left(\frac{m_2}{\Omega_2}\right)^{m_2}\left(\frac{c}{\Omega_3}\right)^{c}$$

$$z^{2m_1-1}\sum_{i=0}^{\infty}(-1)^i\left(\frac{m_1}{\Omega_1}\right)^i \frac{1}{i!}\, z^{2i}$$

$$\int_0^\infty dy\;\, y^{-1-2m_1+2m_2-2i}\, e^{-\frac{m_2}{\Omega_2}y^2}\int_0^\infty dw\;\, w^{-1-2m_1-2i+c}\, e^{-\frac{c}{\Omega_3}w} =$$

$$p_z(z) = \frac{4}{\Gamma(m_1)\Gamma(m_2)\Gamma(c)}\left(\frac{m_1}{\Omega_1}\right)^{m_1}\left(\frac{m_2}{\Omega_2}\right)^{m_2}\left(\frac{c}{\Omega_3}\right)^{c} z^{2m_1-1}$$

$$\sum_{i=0}^{\infty}(-1)^i\left(\frac{m_1}{\Omega_1}\right)^i\frac{1}{i!}\, z^{2i}\,\frac{1}{2}\left(\frac{\Omega_2}{m_2}\right)^{-m_1+m_2-i}\Gamma(-m_1+m_2-i)$$

$$\left(\frac{\Omega_3}{c}\right)^{-2m_1-2i+c}\Gamma\left(-2m_1-2i+c\right) \tag{8}$$

where is $\Gamma(.)$ - Gama function [8], $m_k, k \in \{1,2\}$ - Nakagami-m fading severity, c - Gamma fading severity, and $\Omega_l, l \in \{1,2,3\}$ average power of random process.

By using this PDF it is possible to calculate outage probability (P_o) and channel capacity (C).

Outage probability (OP), standard performance measure that defines the probability that received signal falls under QoS predetermined threshold can be determined according to [9,10]:

$$P_0 = \int_0^{z_0} d \quad p_z(z) = F_z(z_0) \qquad (9)$$

Channel capacity, which defines maximal rate of information at which reliably transmission could be performed, can be determined according to [11]:

$$C = \int_0^{\infty} d \quad (ln(1+z^2)) \quad p_z(z) \qquad (10)$$

NUMERIC RESULTS

Influence of parameters of Nakagami-m and Gamma fading, fading severity and signal power on *PDF* of random variable *z* at the entrance into receiver is displayed graphically on Figure 1 - Figure 3. Numeric results are obtained on basis of derived expression (8).

Figure 1 shows dependency of *PDF* of resulting envelope *z* from Nakagami-m fading severity parameters when Gamma fading severity parameter and signal power are constant.

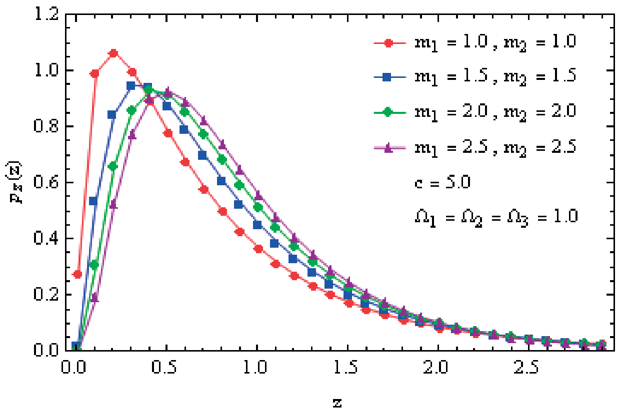

Figure 1. PDF of random variable *z* at the entrance into receiver for different Nakagami-m fading severity when Gamma fading severity and signal power are constant

Figure 2 shows dependency of *PDF* of resulting parameter *z* from Gamma fading severity parameter when Nakagami-m fading severity parameter and signal power are constant.

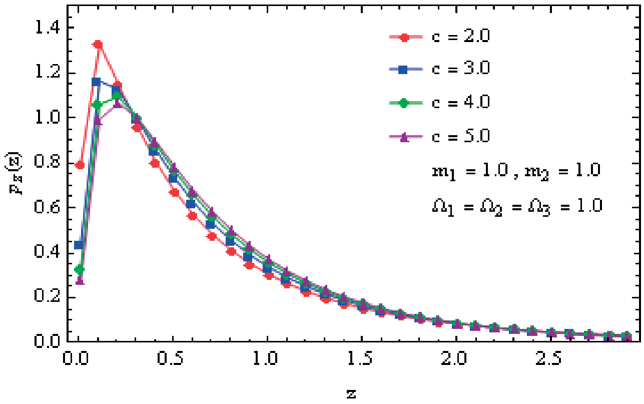

Figure 2. PDF of random variable z at the entrance into receiver for different Gamma fading severity when Nakagami-m fading severity and signal power are constant

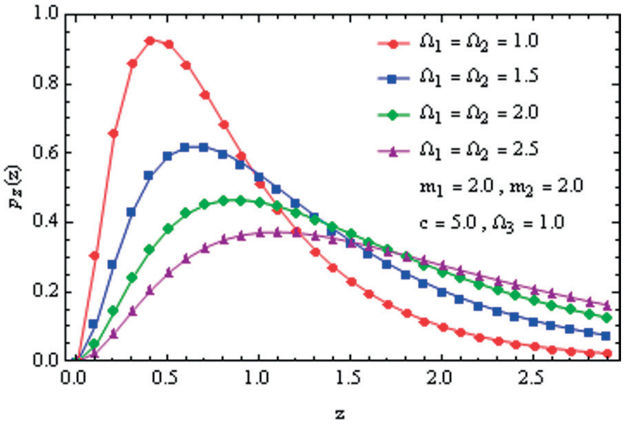

Figure 3. PDF of random variable *z* at the entrance to receiver for different signal powers when Nakagami-m fading severity and signal power are constant

Figure 3 shows dependency of *PDF* of resulting envelope *z* when signal powers under influence of Nakagami-m fading are variables, and Nakagami-m fading severity, Gamma fading severity and signal power under influence of Gamma fading are constant.

Figure 4 shows dependency of OP from threshold for different Nakagami-*m* fading severity when Gamma fading severity and signal power are constant. Numeric results are obtained on basis of expression (9). It can be seen from Figure 4 that better system performances (lower OP values) could be reached in the area of higher m_1 values, namely when desired signal propagates through less severe fading environment.

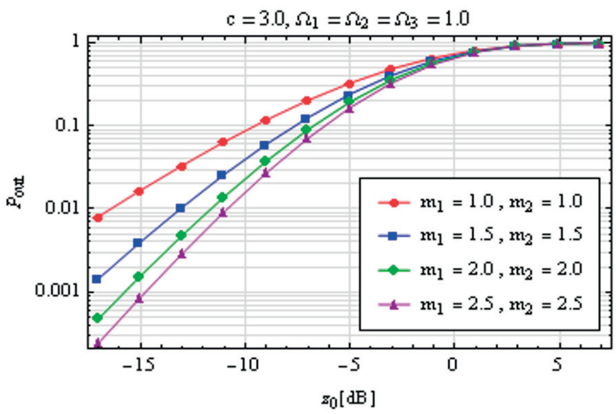

Figure 4. Outage probability for different Nakagami-m fading severity parameter

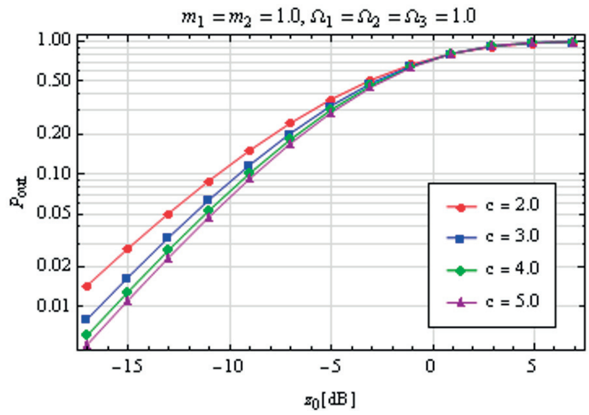

Figure 5. Outage probability depending on threshold for different Gamma fading severity parameter values

Figure 5 shows dependency of OP from threshold for different Gamma fading severity when Nakagami-*m* fading severity and signal power are constant. As expected higher values of parameter *c* provide better system performance values, since the influence of shadowing declines with parameter *c* growth.

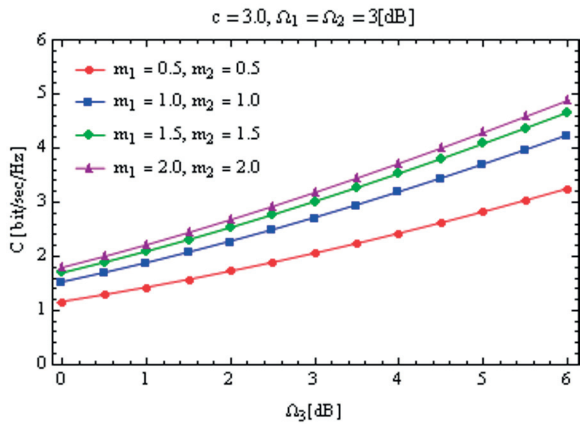

Figure 6. Channel capacity depending on power of Gamma fading for various Nakagami-m fading severity cases

Figure 6 shows functional dependency of channel capacity from power of Gamma fading for different values of Nakagami-m fading severity parameter. It could be seen how channel capacity grows when m_1 parameter values increase.

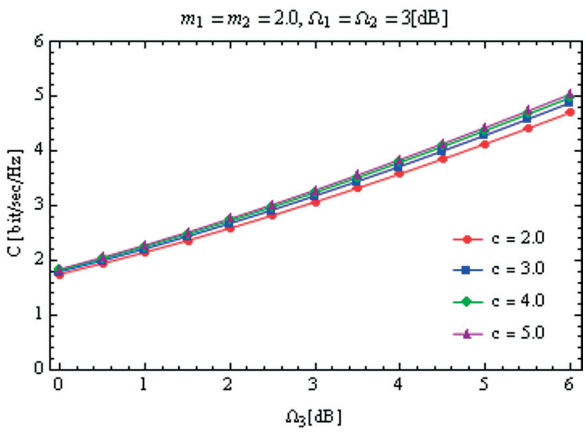

Figure 7. Channel capacity depending on power of Gamma fading for different Gamma fading severity parameter values.

Figure 7 shows dependency of channel capacity from power of Gamma fading for different Gamma fading severity parameter values. With the increase of parameter *c* is visible how channel capacity slightly increase due to shadowing declining.

Acknowledgment

This paper is part of research project No. TR32023 supported by the Ministry of Science and Technological Development of the Republic of Serbia.

CONCLUSION

In this paper for the first time has been considered the case when wireless transmission is exposed to the simultainious influence of two fast fading processes modeled by Nakagami-*m* models and single shadowing proccess modeled with Gamma model. For this case of interest, PDF of resulting envelope proccess has been presented in the closed-form. Further, standard transmission performance criterions, such are outage probability and channel capacity have been efficiantly evaluated and observed in the function of fading and shadowing parameters.

REFERENCES

[1] M. K. Simon, M. S. Alouini, Digital communication over fading channels, John Wiley, New York, 2000.

[2] A. Goldsmith, Wireless communications, Cambridge University Press, New York, 2005.

[3] J. D. Parsons, The mobile radio propagation channel, Pentech Press, London, 1992.

[4] G. L. Stuber, Principles of mobile communication, 2nd ed., Kluwer Academic Publisher, Dordrecht ,2000.

[5] M. Nakagami, "The m-distribution - A general formula of intensity distribution of rapid fading", in Statistical Methods in Radio Wave Propagation, Pergamon Press, Oxford, U.K., pp. 3-36, 1960.

[6] N. Youssef, T. Munakata and M. Takeda, "Fade Statistics in Nakagami Fading Environments", Proc. IEEE 4th Int. Symp. Spread Spectrum Techniques & Applications, pp. 1244-47, Mainz, Germany, 1996.

[7] S. Panic, M. Stefanovic, J. Anastasov and P. Spalevic, "Fading and interference mitigation in wireless communications", CRC Press, New York, 2013.

[8] I. S. Gradshteyn and I. M. Ryzhik, "Table of Integrals, Series, and Products", Academic Press, 5th ed., Orlando, 1994.

[9] S. Panić, P. Spalević, J. Anastasov, M. Stefanović and M. Petrović, "On the performance analysis of SIR-based SSC diversity over correlated α-μ fading channels", Computers and Electrical Engineering, vol. 37, Iss. 3, pp. 332-338, 2011.

[10] P. Spalević, S. Panić, Ć. Dolićanin, M. Stefanović and A. Mosić, "SSC Diversiity Receiver over Correlated α-μ Fading Channels in the Presence of co-channel interference", EURASIP Journal on Wireless Communications and Networking, Volume 2010, Article ID 142392, pp. 1-7

[11] M. Stefanović, J. Anastasov, S. Panić, P. Spalevic and Ć. Dolićanin, "Channel Capacity Analysis Under Various Adaptation Policies and Diversity Techniques over Fading Channels", in Wireless Communications and Networks - Recent Advances, Ali Eksim, Eds. Rijeka: InTech, 2012.